

Package ‘cpfa’

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Type Package

Title Classification with Parallel Factor Analysis

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Description Classification using Richard A. Harshman's Parallel Factor Analysis-1 (Parafac) model or Parallel Factor Analysis-2 (Parafac2) model fit to a three-way or four-way data array. See Harshman and Lundy (1994): <doi:10.1016/0167-9473(94)90132-5>. Uses component weights from one mode of a Parafac or Parafac2 model as features to tune parameters for one or more classification methods via a k-fold cross-validation procedure. Allows for constraints on different tensor modes. Supports penalized logistic regression, support vector machine, random forest, feed-forward neural network, and regularized discriminant analysis. Supports binary and multiclass classification. Predicts class labels or class probabilities and calculates multiple classification performance measures. Implements parallel computing via the 'parallel' and 'doParallel' packages.

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Description

Fits Richard A. Harshman's Parallel Factor Analysis-1 (Parafac) model or Parallel Factor Analysis-2 (Parafac2) model to a three-way or four-way data array. Allows for multiple constraint options on different tensor modes. Uses Parafac component weights from a single mode of this model as predictors to tune parameters for one or more classification methods via a k-fold cross-validation procedure. Predicts class labels and calculates multiple performance measures for binary or multiclass classification over some number of replications with different train-test splits. Provides descriptive statistics to pool output across replications.

Usage

```
cpfa(x, y, model = c("parafac", "parafac2"), nfac = 1,
     nrep = 5, ratio = 0.8, nfolds = 10,
     method = c("PLR", "SVM", "RF", "NN", "RDA"),
     family = c("binomial", "multinomial"), parameters = list(),
     type.out = c("measures", "descriptives"), foldid = NULL,
     prior = NULL, cmode = NULL, seeds = NULL, plot.out = FALSE,
     plot.measures = NULL, parallel = FALSE, cl = NULL,
     verbose = TRUE, ...)
```

Arguments

x	Three-way or four-way data array. Must contain real numbers. See note below.
y	A vector containing at least two unique class labels. Should be a factor that contains two or more levels. For binary case, ensure the order of factor levels (left to right) is such that negative class is first and positive class is second.
model	Character designating the Parafac model to use, either model = "parafac" to fit the Parafac model or model = "parafac2" to fit the Parafac2 model.
nfac	Number of components for each Parafac or Parafac2 model to fit. Default is nfac = 1.
nrep	Number of replications to repeat the procedure. Default is nrep = 5.
ratio	Split ratio for dividing data into train and test sets. Default is ratio = 0.8.
nfolds	Numeric setting number of folds for k-fold cross-validation. Must be 2 or greater. Default is nfolds = 10.
method	Character vector indicating classification methods to use. Possible methods include penalized logistic regression (PLR); support vector machine (SVM); random forest (RF); feed-forward neural network (NN); and regularized discriminant analysis (RDA). If none selected, default is to use all methods with method = c("PLR", "SVM", "RF", "NN", "RDA").

family	Character value specifying binary classification (<code>family = "binomial"</code>) or multiclass classification (<code>family = "multinomial"</code>). If not provided, number of levels of input <code>y</code> is used, where two levels is binary, and where three or more levels is multiclass.
parameters	<p>List containing arguments related to classification methods. When specified, must contain one or more of the following:</p> <p>alpha Values for penalized logistic regression alpha parameter; default is <code>alpha = seq(0, 1, length = 6)</code>. Must be numeric and contain only real numbers between 0 and 1, inclusive.</p> <p>lambda Optional user-supplied lambda sequence for <code>cv.glmnet</code> for penalized logistic regression. Default is <code>NULL</code>.</p> <p>cost Values for support vector machine cost parameter; default is <code>cost = c(1, 2, 4, 8, 16, 32, 64)</code>. Must be numeric and contain only real numbers greater than or equal to zero.</p> <p>gamma Values for support vector machine gamma parameter; default is <code>gamma = c(0, 0.01, 0.1, 1, 10, 100, 1000)</code>. Must be numeric and greater than or equal to 0.</p> <p>ntree Values for random forest number of trees parameter; default is <code>ntree = c(100, 200, 400, 600, 800, 1600, 3200)</code>. Must be numeric and contain only integers greater than or equal to 1.</p> <p>nodesize Values for random forest node size parameter; default is <code>nodesize = c(1, 2, 4, 8, 16, 32, 64)</code>. Must be numeric and contain only integers greater than or equal to 1.</p> <p>size Values for neural network size parameter; default is <code>size = c(1, 2, 4, 8, 16, 32, 64)</code>. Must be numeric and contain only integers greater than or equal to 0.</p> <p>decay Values for neural network decay parameter; default is <code>decay = c(0.001, 0.01, 0.1, 1, 2, 4, 8, 16)</code>. Must be numeric and contain only real numbers.</p> <p>rda.alpha Values for regularized discriminant analysis alpha parameter; default is <code>rda.alpha = seq(0, 0.999, length = 6)</code>. Must be numeric and contain only real numbers between 0 (inclusive) and 1 (exclusive).</p> <p>delta Values for regularized discriminant analysis delta parameter; default is <code>delta = c(0, 0.1, 1, 2, 3, 4)</code>. Must be numeric and contain only real numbers greater than or equal to 0.</p>
type.out	Type of output desired: <code>type.out = "measures"</code> gives array containing classification performance measures for all replications while <code>type.out = "descriptives"</code> gives list of descriptive statistics calculated across all replications for each performance measure. Both options also provide the estimated training weights and classification weights. Defaults to <code>type.out = "descriptives"</code> .
foldid	Integer vector containing fold IDs for k-fold cross-validation. If not provided, fold IDs are generated randomly for number of folds <code>nfolds</code> .
prior	Prior probabilities of class membership. If unspecified, the class proportions for input <code>y</code> are used. If specified, the probabilities should be in the order of the factor levels of input <code>y</code> .

<code>cmode</code>	Integer value of 1, 2, or 3 (or 4 if <code>x</code> is a four-way array) specifying the mode whose component weights will be predictors for classification. Defaults to the last mode of the inputted array (i.e., defaults to 3 for three-way array, and to 4 for four-way array). If <code>model = "parafac2"</code> , last mode will be used.
<code>seeds</code>	Random seeds to be associated with each replication. Default is <code>seeds = 1:nrep</code> .
<code>plot.out</code>	Logical indicating whether to output one or more box plots of classification performance measures that are plotted across classification methods and number of components.
<code>plot.measures</code>	Character vector containing values that specify for plotting one or more of 11 possible classification performance measures. Only relevant when <code>plot.out = TRUE</code> . Should contain one or more of the following labels: <code>c("err", "acc", "tpr", "fpr", "tnr", "fnr", "ppv", "npv", "fdr", "fom", "fs")</code> . A box plot will be created for each measure that is specified, summarizing output across replications. Note that additional information about each label is available in the Details section of the help file for function <code>cpm</code> . Note also that there are a few cases where the x-axis tick labels for a plot might not appear. This issue will be resolved in a future update.
<code>parallel</code>	Logical indicating if parallel computing should be implemented. If <code>TRUE</code> , the package parallel is used for parallel computing. For all classification methods except penalized logistic regression, the doParallel package is used as a wrapper. Defaults to <code>FALSE</code> , which implements sequential computing.
<code>cl</code>	Cluster for parallel computing, which is used when <code>parallel = TRUE</code> . Note that if <code>parallel = TRUE</code> and <code>cl = NULL</code> , then the cluster is defined as <code>makeCluster(detectCores())</code> .
<code>verbose</code>	If <code>TRUE</code> , progress is printed.
<code>...</code>	Additional arguments to be passed to function <code>parafac</code> for fitting a Parafac model or function <code>parafac2</code> for fitting a Parafac2 model. Example: can impose different constraints on different modes of the input array using the argument <code>const</code> . See help file for function <code>parafac</code> or for function <code>parafac2</code> for additional details.

Details

After fitting a Parafac or Parafac2 model with package **multiway** (see `parafac` or `parafac2` in **multiway** for details), the estimated classification mode weight matrix is passed to one or several of five classification methods—including penalized logistic regression (PLR); support vector machine (SVM); random forest (RF); feed-forward neural network (NN); and regularized discriminant analysis (RDA).

Package **glmnet** fits models for PLR. PLR tunes penalty parameter λ while the elastic net parameter α is set by the user (see the help file for function `cv.glmnet` in package **glmnet**). For SVM, package **e1071** is used with a radial basis kernel. Penalty parameter `cost` and radial basis parameter `gamma` are used (see `svm` in package **e1071**). For RF, package **randomForest** is used and implements Breiman's random forest algorithm. The number of predictors sampled at each node split is set at the default of `sqrt(R)`, where `R` is the number of Parafac or Parafac2 components. Two tuning parameters allowed are `ntree`, the number of trees to be grown, and `nodesize`, the minimum size of terminal nodes (see `randomForest` in package **randomForest**). For NN, package **nnet** fits a single-hidden-layer, feed-forward neural network model. Penalty parameters `size` (i.e., number of

hidden layer units) and decay (i.e., weight decay) are used (see **nnet**). For RDA, package **rda** fits a shrunken centroids regularized discriminant analysis model. Tuning parameters include `rda.alpha`, the shrinkage penalty for the within-class covariance matrix, and `delta`, the shrinkage penalty of class centroids towards the overall dataset centroid.

For all five methods, k-fold cross-validation is implemented to tune classification parameters where the number of folds is set by argument `nfolds`. After tuning, class labels are predicted using optimal parameters; and classification performance measures are calculated. The process is repeated over a number of replications with different random splits of the input array and of the class labels at each replication.

Value

Returns either a three-way array with classification performance measures for each model and for each replication, or a list containing matrices with different descriptive statistics for performance measures calculated across all replications. Specify `type.out = "measures"` to output the array of performance measures. Specify `type.out = "descriptives"` to output descriptive statistics across replications. In addition, for both options, the following objects are also provided:

<code>predweights</code>	List of predicted classification weights for each Parafac or Parafac2 model and for each replication.
<code>train.weights</code>	List of lists of training weights for each Parafac or Parafac2 model and for each replication.
<code>opt.tune</code>	List of optimal tuning parameters for classification methods for each Parafac or Parafac2 model and for each replication.
<code>mean.opt.tune</code>	Mean across all replications of optimal tuning parameters for classification methods for each Parafac or Parafac2 model.

Note

If argument `cmode` is not null, input array `x` is reshaped with function `aperm` such that the `cmode` dimension of `x` is ordered last. Estimated mode A and B (and mode C for a four-way array) weights that are outputted as `Aweights` and `Bweights` (and `Cweights`) reflect this permutation. For example, if `x` is a four-way array and `cmode = 2`, the original input modes 1, 2, 3, and 4 will correspond to output modes 1, 3, 4, 2. Here, output A = input 1; B = 3, and C = 4 (i.e., the second mode specified by `cmode` has been moved to the D mode/last mode). For `model = "parafac2"`, classification mode is assumed to be the last mode (i.e., mode C for three-way array and mode D for four-way array).

In addition, note that the following combination of arguments will give an error: `nfac = 1`, `family = "multinomial"`, `method = "PLR"`. The issue arises from providing `glmnet::cv.glmnet` input `x` a matrix with a single column. The issue is resolved for `family = "binomial"` because a column of 0s is appended to the single column, but this solution does not appear to work for the multiclass case. As such, this combination of arguments is not currently allowed. This issue will be resolved in a future update.

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Examples

```
##### Parafac example with 3-way array and binary response #####

# set seed and specify dimensions of a three-way tensor
set.seed(3)
mydim <- c(10, 11, 100)
nf <- 3

# create correlation matrix between response and third mode's weights
rho.cc <- .35
rho.cy <- .75
```

```

cormat.values <- c(1, rho.cc, rho.cc, rho.cy, rho.cc, 1, rho.cc, rho.cy,
                  rho.cc, rho.cc, 1, rho.cy, rho.cy, rho.cy, rho.cy, 1)
cormat <- matrix(cormat.values, nrow = (nf + 1), ncol = (nf + 1))

# sample from a multivariate normal with specified correlation structure
ymean <- Cmean <- 2
mu <- as.matrix(c(Cmean, Cmean, Cmean, ymean))
eidecomp <- eigen(cormat, symmetric = TRUE)
L.sqrt <- diag(eidecomp$values^0.5)
cormat.sqrt <- eidecomp$vectors %*% L.sqrt %*% t(eidecomp$vectors)
Z <- matrix(rnorm(mydim[3] * (nf + 1)), nrow = mydim[3], ncol = (nf + 1))
Xw <- rep(1, mydim[3]) %*% t(mu) + Z %*% cormat.sqrt
Cmat <- Xw[, 1:nf]

# create a random three-way data tensor with C weights related to a response
Amat <- matrix(rnorm(mydim[1] * nf), nrow = mydim[1], ncol = nf)
Bmat <- matrix(runif(mydim[2] * nf), nrow = mydim[2], ncol = nf)
Xmat <- tcrossprod(Amat, krprod(Cmat, Bmat))
Xmat <- array(Xmat, dim = mydim)
Emat <- array(rnorm(prod(mydim)), dim = mydim)
Emat <- nscale(Emat, 0, ssnew = sumsq(Xmat))
X <- Xmat + Emat

# create a binary response by dichotomizing at the specified response mean
y <- factor(as.numeric(Xw[, (nf + 1)] > ymean))

# initialize
alpha <- seq(0, 1, length = 2)
gamma <- c(0, 0.01)
cost <- c(1, 2)
ntree <- c(100, 200)
nodesize <- c(1, 2)
size <- c(1, 2)
decay <- c(0, 1)
rda.alpha <- c(0.1, 0.6)
delta <- c(0.1, 2)
method <- c("PLR", "SVM", "RF", "NN", "RDA")
family <- "binomial"
parameters <- list(alpha = alpha, gamma = gamma, cost = cost, ntree = ntree,
                  nodesize = nodesize, size = size, decay = decay,
                  rda.alpha = rda.alpha, delta = delta)

model <- "parafac"
nfolds <- 3

# constrain first mode weights to be orthogonal
const <- c("orthog", "uncons", "uncons")

# fit Parafac models and use third mode weights to tune classification
# methods, to predict class labels, and to return classification
# performance measures pooled across multiple train-test splits
output <- cpfa(x = X, y = y, model = model, nfac = 3, nrep = 2, ratio = 0.8,
              nfolds = nfolds, method = method, family = family,
              parameters = parameters, type.out = "descriptives",

```

```

seeds = NULL, plot.out = TRUE, parallel = FALSE, const = const)

# print performance measure means across train-test splits
output$descriptive$mean

##### Parafac2 example with 4-way array and multiclass response #####

# set seed and specify dimensions of a four-way tensor
set.seed(5)
mydim <- c(10, 11, 12, 100)
nf <- 3

# create correlation matrix between response and fourth mode's weights
rho.dd <- .35
rho.dy <- .75
cormat.values <- c(1, rho.dd, rho.dd, rho.dy, rho.dd, 1, rho.dd, rho.dy,
                  rho.dd, rho.dd, 1, rho.dy, rho.dy, rho.dy, rho.dy, 1)
cormat <- matrix(cormat.values, nrow = (nf + 1), ncol = (nf + 1))

# sample from a multivariate normal with specified correlation structure
ymean <- Dmean <- 2
mu <- as.matrix(c(Dmean, Dmean, Dmean, ymean))
eidecomp <- eigen(cormat, symmetric = TRUE)
L.sqrt <- diag(eidecomp$values^0.5)
cormat.sqrt <- eidecomp$vectors %*% L.sqrt %*% t(eidecomp$vectors)
Z <- matrix(rnorm(mydim[4] * (nf + 1)), nrow = mydim[4], ncol = (nf + 1))
Xw <- rep(1, mydim[4]) %*% t(mu) + Z %*% cormat.sqrt
Dmat <- Xw[, 1:nf]

# create a random four-way data tensor with D weights related to a response
Bmat <- matrix(runif(mydim[2] * nf), nrow = mydim[2], ncol = nf)
Cmat <- matrix(runif(mydim[3] * nf), nrow = mydim[3], ncol = nf)
nDd <- rep(c(10, 12, 14), length.out = mydim[4])
Gmat <- matrix(rnorm(nf * nf), nrow = nf)
Amat <- vector("list", mydim[4])
X <- Xmat <- Emat <- Amat
for (Dd in 1:mydim[4]) {
  Amat[[Dd]] <- matrix(nf * rnorm(nDd[Dd]), nrow = nDd[Dd], ncol = nf)
  Amat[[Dd]] <- svd(Amat[[Dd]], nv = 0)$u %*% Gmat
  leftMat <- Amat[[Dd]] %*% diag(Dmat[Dd,])
  Xmat[[Dd]] <- array(tcrossprod(leftMat, krprod(Cmat, Bmat)),
                    dim = c(nDd[Dd], mydim[2], mydim[3]))
  Emat[[Dd]] <- array(rnorm(nDd[Dd] * mydim[2] * mydim[3]),
                    dim = c(nDd[Dd], mydim[2], mydim[3]))
  X[[Dd]] <- Xmat[[Dd]] + Emat[[Dd]]
}

# create a multiclass response
stor <- matrix(rep(1, nrow(Xw)), nrow = nrow(Xw))
stor[which(Xw[, (nf + 1)] < (ymean - 0.4 * sd(Xw[, (nf + 1)])))] <- 2
stor[which(Xw[, (nf + 1)] > (ymean + 0.4 * sd(Xw[, (nf + 1)])))] <- 0
y <- factor(stor)

```



```

# initialize
alpha <- seq(0, 1, length = 2)
gamma <- c(0, 1)
cost <- c(0.1, 5)
ntree <- c(200, 300)
nodesize <- c(1, 2)
size <- c(1, 2)
decay <- c(0, 1)
rda.alpha <- seq(0.1, 0.9, length = 2)
delta <- c(0.1, 1)
method <- c("PLR", "SVM", "RF", "NN", "RDA")
family <- "multinomial"
parameters <- list(alpha = alpha, gamma = gamma, cost = cost, ntree = ntree,
                  nodesize = nodesize, size = size, decay = decay,
                  rda.alpha = rda.alpha, delta = delta)
model <- "parafac2"
nfolds <- 3

# constrain first mode weights to be orthogonal, fourth mode to be nonnegative
const <- c("orthog", "uncons", "uncons", "nonneg")

# fit Parafac2 model and use fourth mode weights to tune classification
# methods, to predict class labels, and to return classification
# performance measures pooled across multiple train-test splits
output <- cpfa(x = X, y = y, model = model, nfac = nf, nrep = 2, ratio = 0.8,
              nfolds = nfolds, method = method, family = family,
              parameters = parameters, type.out = "descriptives",
              seeds = NULL, plot.out = TRUE, parallel = FALSE, const = const)

# print performance measure means across train-test splits
output$descriptive$mean

```

cpm

Classification Performance Measures

Description

Calculates multiple performance measures for binary or multiclass classification. Uses known class labels and evaluates against predicted labels.

Usage

```
cpm(x, y, level = NULL, fbeta = NULL, prior = NULL)
```

Arguments

x Known class labels of class numeric, factor, or integer. If factor, converted to class integer in the order of factor levels with integers beginning at 0 (i.e., for binary classification, factor levels become 0 and 1; for multiclass, levels become 0, 1, 2, etc.).

<code>y</code>	Predicted class labels of class numeric, factor, or integer. If factor, converted to class integer in the order of factor levels with integers beginning at 0 (i.e., for binary classification, factor levels become 0 and 1; for multiclass, 0, 1, 2, etc.).
<code>level</code>	Optional argument specifying possible class labels. For cases when <code>x</code> or <code>y</code> do not contain all possible classes. Can be of class numeric, integer, or character. Must contain two elements for binary classification, and contain three or more elements for multiclass classification. If integer, integers should be ordered (e.g., binary with <code>c(0, 1)</code> ; or three-class with <code>c(0, 1, 2)</code>). Note: if both <code>x</code> and <code>y</code> jointly contain only a single value (e.g., 1), must specify argument <code>level</code> in order to identify classification as binary or multiclass.
<code>fbeta</code>	Optional numeric argument specifying beta value for F-score. Defaults to <code>fbeta = 1</code> , providing an F1-score (i.e., the balanced harmonic mean between precision and recall). Can be any real number.
<code>prior</code>	Optional numeric argument specifying weights for classes. Currently only implemented with multiclass problems. Defaults to <code>prior = c(rep(1/llev, llev))</code> , where <code>llev</code> is the number of classes, providing equal importance across classes.

Details

Selecting one class as a negative class and one class as a positive class, binary classification generates four possible outcomes: (1) negative cases classified as positives, called false positives (FP); (2) negative cases classified as negatives, called true negatives (TN); (3) positive cases classified as negatives, called false negatives (FN); and (4) positive cases classified as positives, called true positives (TP).

Multiple evaluation measures are calculated using these four outcomes. Measures include: overall error (ERR), also called fraction incorrect; overall accuracy (ACC), also called fraction correct; true positive rate (TPR), also called recall, hit rate, or sensitivity; false negative rate (FNR), also called miss rate; false positive rate (FPR), also called fall-out; true negative rate (TNR), also called specificity or selectivity; positive predictive value (PPV), also called precision; false discovery rate (FDR); negative predictive value (NPV); false omission rate (FOR); and F-score (FS).

In multiclass classification, the four outcomes are possible for each individual class in macro-averaging, and performance measures are averaged over classes. Macro-averaging gives equal importance to all classes. For multiclass classification, calculated measures are currently only macro-averaged. See the listed reference in this help file for additional details on micro-averaging.

Note that, for binary classification, this function assumes a negative class and a positive class (i.e., it contains a reference group) and is ordered. Multiclass classification is currently assumed to be unordered.

Computational details:

$$\text{ERR} = (\text{FP} + \text{FN}) / (\text{TP} + \text{TN} + \text{FP} + \text{FN}).$$

$$\text{ACC} = (\text{TP} + \text{TN}) / (\text{TP} + \text{TN} + \text{FP} + \text{FN}), \text{ and } \text{ACC} = 1 - \text{ERR}.$$

$$\text{TPR} = \text{TP} / (\text{TP} + \text{FN}).$$

$$\text{FNR} = \text{FN} / (\text{FN} + \text{TP}), \text{ and } \text{FNR} = 1 - \text{TPR}.$$

$$\text{FPR} = \text{FP} / (\text{FP} + \text{TN}).$$

$$\text{TNR} = \text{TN} / (\text{TN} + \text{FP}), \text{ and } \text{TNR} = 1 - \text{FPR}.$$

$$\text{PPV} = \text{TP} / (\text{TP} + \text{FP}).$$

$$\text{FDR} = \text{FP} / (\text{FP} + \text{TP}), \text{ and } \text{FDR} = 1 - \text{PPV}.$$

$$\text{NPV} = \text{TN} / (\text{TN} + \text{FN}).$$

$$\text{FOR} = \text{FN} / (\text{FN} + \text{TN}), \text{ and } \text{FOR} = 1 - \text{NPV}.$$

$$\text{FS} = (1 + \text{beta}^2) * ((\text{PPV} * \text{TPR}) / (((\text{beta}^2) * \text{PPV}) + \text{TPR})).$$

All performance measures calculated are between 0 and 1, inclusive. For multiclass classification, macro-averaged values are provided for each performance measure. Note that 'beta' in FS represents the relative weight such that recall (TPR) is beta times more important than precision (PPV). See reference for more details.

Value

Returns list where first element is a full confusion matrix `cm` and where the second element is a data frame containing performance measures. Note that for multiclass classification, macro-averaged values are provided (i.e., each measure is calculated for each class, then averaged over all classes; the average is weighted by argument `prior` if provided). The second list element contains the following performance measures:

<code>cm</code>	A confusion matrix with counts for each of the possible outcomes.
<code>err</code>	Overall error (ERR). Also called fraction incorrect.
<code>acc</code>	Overall accuracy (ACC). Also called fraction correct.
<code>tp</code>	True positive rate (TPR). Also called recall, hit rate, or sensitivity.
<code>fpr</code>	False positive rate (FPR). Also called fall-out.
<code>tnr</code>	True negative rate (TNR). Also called specificity or selectivity.
<code>fnr</code>	False negative rate (FNR). Also called miss rate.
<code>ppv</code>	Positive predictive value (PPV). Also called precision.
<code>npv</code>	Negative predicted value (NPV).
<code>fdr</code>	False discovery rate (FDR).
<code>fom</code>	False omission rate (FOR).
<code>fs</code>	F-score. Mean between TPR (recall) and PPV (precision) varying by importance given to recall over precision (see Details section and argument <code>fbeta</code>).

Author(s)

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References

Sokolova, M. and Lapalme, G. (2009). A systematic analysis of performance measures for classification tasks. *Information Processing and Management*, 45(4), 427-437.


```

model <- "parafac"
nfolds <- 3

# constrain first mode weights to be orthogonal
const <- c("orthog", "uncons", "uncons")

# fit Parafac models and use third mode to tune classification methods
tune.object <- tune.cpfa(x = X, y = y, model = model, nfac = nf,
                        nfolds = nfolds, method = method, family = family,
                        parameters = parameters, parallel = FALSE,
                        const = const)

# create new data with Parafac structure and C weights related to response
mydim.new <- c(10, 11, 20)
Znew <- matrix(rnorm(mydim.new[3]*(nf + 1)),
              nrow = mydim.new[3], ncol = (nf + 1))
Xwnew <- rep(1, mydim.new[3]) %*% t(mu) + Znew %*% cormat.sqrt
Cmatnew <- Xwnew[, 1:nf]
Xnew0 <- tcrossprod(Amat, krprod(Cmatnew, Bmat))
Xnew0 <- array(Xnew0, dim = mydim.new)
Ematnew <- array(rnorm(prod(mydim.new)), dim = mydim.new)
Ematnew <- nscale(Ematnew, 0, ssnew = sumsq(Xnew0))
Xnew <- Xnew0 + Ematnew

# create new random class labels for two levels
newlabel <- as.numeric(Xwnew[, (nf + 1)] > ymean)

# predict class labels
predict.labels <- predict(object = tune.object, newdata = Xnew,
                          type = "response")

# calculate performance measures for predicted class labels
y.pred <- predict.labels[, 1]
evalmeasure <- cpm(x = newlabel, y = y.pred)

# print performance measures
evalmeasure

```

cpm.all

Wrapper for Calculating Classification Performance Measures

Description

Applies function `cpm` to multiple sets of class labels. Each set of class labels is evaluated against the same set of predicted labels. Works with output from function `predict.cpfa` and calculates classification performance measures for multiple classifiers or numbers of components.

Usage

```
cpm.all(x, y, ...)
```

Arguments

x	A data frame where each column contains a set of known class labels of class numeric, factor, or integer. If a set is of class factor, that set is converted to class integer in the order of factor levels with integers beginning at 0 (i.e., for binary classification, factor levels become 0 and 1; for multiclass, levels become 0, 1, 2, etc.).
y	Predicted class labels of class numeric, factor, or integer. If factor, converted to class integer in order of factor levels with integers beginning at 0 (i.e., for binary classification, factor levels become 0 and 1; for multiclass, 0, 1, 2, etc.).
...	Additional arguments to be passed to function cpm for calculating classification performance measures.

Details

Wrapper function that applies function cpm to multiple sets of class labels and one set of predicted labels. See help file for function cpm for additional details.

Value

Returns a list with the following two elements:

cm.list	A list of confusion matrices, denoted cm, where each confusion matrix is associated with one comparison.
cpms	A data frame containing classification performance measures where each row contains measures for one comparison.

Author(s)

Matthew Snodgrass <snodg031@umn.edu>

References

Sokolova, M. and Lapalme, G. (2009). A systematic analysis of performance measures for classification tasks. *Information Processing and Management*, 45(4), 427-437.

Examples

```
##### Parafac example with 3-way array and binary response #####

# set seed and specify dimensions of a three-way tensor
set.seed(3)
mydim <- c(10, 11, 80)
nf <- 3

# create correlation matrix between response and third mode's weights
rho.cc <- .35
rho.cy <- .75
cormat.values <- c(1, rho.cc, rho.cc, rho.cy, rho.cc, 1, rho.cc, rho.cy,
                  rho.cc, rho.cc, 1, rho.cy, rho.cy, rho.cy, rho.cy, 1)
cormat <- matrix(cormat.values, nrow = (nf + 1), ncol = (nf + 1))
```

```

# sample from a multivariate normal with specified correlation structure
ymean <- Cmean <- 2
mu <- as.matrix(c(Cmean, Cmean, Cmean, ymean))
eidecomp <- eigen(cormat, symmetric = TRUE)
L.sqrt <- diag(eidecomp$values^0.5)
cormat.sqrt <- eidecomp$vectors %*% L.sqrt %*% t(eidecomp$vectors)
Z <- matrix(rnorm(mydim[3] * (nf + 1)), nrow = mydim[3], ncol = (nf + 1))
Xw <- rep(1, mydim[3]) %*% t(mu) + Z %*% cormat.sqrt
Cmat <- Xw[, 1:nf]

# create a random three-way data tensor with C weights related to a response
Amat <- matrix(rnorm(mydim[1] * nf), nrow = mydim[1], ncol = nf)
Bmat <- matrix(runif(mydim[2] * nf), nrow = mydim[2], ncol = nf)
Xmat <- tcrossprod(Amat, krprod(Cmat, Bmat))
Xmat <- array(Xmat, dim = mydim)
Emat <- array(rnorm(prod(mydim)), dim = mydim)
Emat <- nscale(Emat, 0, ssnew = sumsq(Xmat))
X <- Xmat + Emat

# create a binary response by dichotomizing at the specified response mean
y <- factor(as.numeric(Xw[, (nf + 1)] > ymean))

# initialize
alpha <- seq(0, 1, length = 2)
gamma <- c(0, 0.01)
cost <- c(1, 2)
ntree <- c(100, 200)
nodesize <- c(1, 2)
size <- c(1, 2)
decay <- c(0, 1)
rda.alpha <- c(0.1, 0.6)
delta <- c(0.1, 2)
method <- c("PLR", "SVM", "RF", "NN", "RDA")
family <- "binomial"
parameters <- list(alpha = alpha, gamma = gamma, cost = cost, ntree = ntree,
                  nodesize = nodesize, size = size, decay = decay,
                  rda.alpha = rda.alpha, delta = delta)

model <- "parafac"
nfolds <- 3

# constrain first mode weights to be orthogonal
const <- c("orthog", "uncons", "uncons")

# fit Parafac models and use third mode to tune classification methods
tune.object <- tune.cpfa(x = X, y = y, model = model, nfac = nf,
                      nfolds = nfolds, method = method, family = family,
                      parameters = parameters, parallel = FALSE,
                      const = const)

# create new data with Parafac structure and C weights related to response
mydim.new <- c(10, 11, 20)
Znew <- matrix(rnorm(mydim.new[3] * (nf + 1)),

```

```

      nrow = mydim.new[3], ncol = (nf + 1))
Xwnew <- rep(1, mydim.new[3]) %*% t(mu) + Znew %*% cormat.sqrt
Cmatnew <- Xwnew[, 1:nf]
Xnew0 <- tcrossprod(Amat, krprod(Cmatnew, Bmat))
Xnew0 <- array(Xnew0, dim = mydim.new)
Ematnew <- array(rnorm(prod(mydim.new)), dim = mydim.new)
Ematnew <- nscale(Ematnew, 0, ssnew = sumsq(Xnew0))
Xnew <- Xnew0 + Ematnew

# create new random class labels for two levels
newlabel <- as.numeric(Xwnew[, (nf + 1)] > ymean)

# predict class labels
predict.labels <- predict(object = tune.object, newdata = Xnew,
                          type = "response")

# calculate performance measures for predicted class labels
evalmeasure <- cpm.all(x = predict.labels, y = newlabel)

# print performance measures
evalmeasure

```

predict.cdfa

Predict Method for Classification with Parallel Factor Analysis

Description

Obtains predictions for class labels from a 'cpfa' model object obtained using function `tune.cdfa`.

Usage

```

## S3 method for class 'cpfa'
predict(object, newdata = NULL, method = NULL,
        type = c("response", "prob", "classify.weights"),
        threshold = NULL, ...)

```

Arguments

<code>object</code>	A fit object of class 'cpfa' from function <code>tune.cdfa</code> .
<code>newdata</code>	An optional three-way or four-way data array used to predict Parafac or Parafac2 component weights using estimated Parafac or Parafac2 model component weights from inputted object. Dimensions must match dimensions of original data for all modes except the classification mode. If omitted, the original data are used.
<code>method</code>	Character vector indicating classification methods to use. Possible methods include penalized logistic regression (PLR); support vector machine (SVM); random forest (RF); feed-forward neural network (NN); and regularized discriminant analysis (RDA). If none selected, default is to use all methods.

type	Character vector indicating type of prediction to return. Possible values include: (1) "response", returning predicted class labels; (2) "prob", returning predicted class probabilities; or (3) "classify.weights", returning predicted component weights used in classification from Parafac models specified. Defaults to "response".
threshold	For binary classification, value indicating prediction threshold over which observations are classified as the positive class. If not provided, calculates threshold using class proportions in original data. For multiclass classification, threshold is not currently implemented.
...	Currently ignored. Additional predict arguments.

Details

Predicts class labels for a binary or a multiclass outcome. Specifically, predicts component weights for one mode of a Parallel Factor Analysis-1 (Parafac) model or a Parallel Factor Analysis-2 (Parafac2) model using new data and previously estimated mode weights from original data. Passes predicted component weights to one or several classification methods as new data for predicting class labels.

Tuning parameters optimized by k-fold cross-validation are used for each classification method (see help for `tune.cdfa`). If not supplied in argument `threshold`, prediction threshold for all classification methods is calculated using proportions of class labels for original data in the binary case (and the positive class proportion is set as the threshold). For multiclass case, class with highest probability is chosen.

Value

Returns one of the following, depending on the choice for argument `type`:

`type = "response"`

A data frame containing predicted class labels or probabilities (binary case) for each Parafac model and classification method selected (see argument `type`). Number of columns is equal to number of methods times number of Parafac models. Number of rows is equal to number of predicted observations.

`type = "prob"`

A list containing predicted probabilities for each Parafac model and classification method selected (see argument `type`). Only returned if original response was multiclass (i.e., contained three or more class labels). The number of list elements is equal to number of methods times the number of Parafac models.

`type = "classify.weights"`

List containing predicted component weights for each Parafac or Parafac2 model. Length is equal to number of Parafac models that were fit.

Author(s)

Matthew Snodgrass <snodg031@umn.edu>

References

- Breiman, L. (2001). Random forests. *Machine Learning*, 45(1), 5-32.
 Cortes, C. and Vapnik, V. (1995). Support-vector networks. *Machine Learning*, 20(3), 273-297.

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- Zou, H. and Hastie, T. (2005). Regularization and variable selection via the elastic net. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 67(2), 301-320.

Examples

```
##### Parafac example with 3-way array and binary response #####

# set seed and specify dimensions of a three-way tensor
set.seed(3)
mydim <- c(10, 11, 80)
nf <- 3

# create correlation matrix between response and third mode's weights
rho.cc <- .35
rho.cy <- .75
cormat.values <- c(1, rho.cc, rho.cc, rho.cy, rho.cc, 1, rho.cc, rho.cy,
                  rho.cc, rho.cc, 1, rho.cy, rho.cy, rho.cy, rho.cy, 1)
cormat <- matrix(cormat.values, nrow = (nf + 1), ncol = (nf + 1))

# sample from a multivariate normal with specified correlation structure
```

```

ymean <- Cmean <- 2
mu <- as.matrix(c(Cmean, Cmean, Cmean, ymean))
eidecomp <- eigen(cormat, symmetric = TRUE)
L.sqrt <- diag(eidecomp$values^0.5)
cormat.sqrt <- eidecomp$vectors %*% L.sqrt %*% t(eidecomp$vectors)
Z <- matrix(rnorm(mydim[3]*(nf + 1)), nrow = mydim[3], ncol = (nf + 1))
Xw <- rep(1, mydim[3]) %*% t(mu) + Z %*% cormat.sqrt
Cmat <- Xw[, 1:nf]

# create a random three-way data tensor with C weights related to a response
Amat <- matrix(rnorm(mydim[1]*nf), nrow = mydim[1], ncol = nf)
Bmat <- matrix(runif(mydim[2]*nf), nrow = mydim[2], ncol = nf)
Xmat <- tcrossprod(Amat, krprod(Cmat, Bmat))
Xmat <- array(Xmat, dim = mydim)
Emat <- array(rnorm(prod(mydim)), dim = mydim)
Emat <- nscale(Emat, 0, ssnew = sumsq(Xmat))
X <- Xmat + Emat

# create a binary response by dichotomizing at the specified response mean
y <- factor(as.numeric(Xw[, (nf + 1)] > ymean))

# initialize
alpha <- seq(0, 1, length = 2)
gamma <- c(0, 0.01)
cost <- c(1, 2)
ntree <- c(100, 200)
nodesize <- c(1, 2)
size <- c(1, 2)
decay <- c(0, 1)
rda.alpha <- c(0.1, 0.6)
delta <- c(0.1, 2)
method <- c("PLR", "SVM", "RF", "NN", "RDA")
family <- "binomial"
parameters <- list(alpha = alpha, gamma = gamma, cost = cost, ntree = ntree,
                  nodesize = nodesize, size = size, decay = decay,
                  rda.alpha = rda.alpha, delta = delta)

model <- "parafac"
nfolds <- 3

# constrain first mode weights to be orthogonal
const <- c("orthog", "uncons", "uncons")

# fit Parafac models and use third mode to tune classification methods
tune.object <- tune.cdfa(x = X, y = y, model = model, nfac = nf,
                      nfolds = nfolds, method = method, family = family,
                      parameters = parameters, parallel = FALSE,
                      const = const)

# create new data with Parafac structure and C weights related to response
mydim.new <- c(10, 11, 20)
Znew <- matrix(rnorm(mydim.new[3]*(nf + 1)),
              nrow = mydim.new[3], ncol = (nf + 1))
Xwnew <- rep(1, mydim.new[3]) %*% t(mu) + Znew %*% cormat.sqrt

```

```

Cmatnew <- Xwnew[, 1:nf]
Xnew0 <- tcrossprod(Amat, krprod(Cmatnew, Bmat))
Xnew0 <- array(Xnew0, dim = mydim.new)
Ematnew <- array(rnorm(prod(mydim.new)), dim = mydim.new)
Ematnew <- nscale(Ematnew, 0, ssnew = sumsq(Xnew0))
Xnew <- Xnew0 + Ematnew

# predict class labels
predict.labels <- predict(object = tune.object, newdata = Xnew,
                          type = "response")

# print predicted labels
predict.labels

##### Parafac2 example with 4-way array and multiclass response #####

# set seed and specify dimensions of a four-way tensor
set.seed(5)
mydim <- c(10, 11, 12, 90)
nf <- 3

# create correlation matrix between response and fourth mode's weights
rho.dd <- .35
rho.dy <- .75
cormat.values <- c(1, rho.dd, rho.dd, rho.dy, rho.dd, 1, rho.dd, rho.dy,
                  rho.dd, rho.dd, 1, rho.dy, rho.dy, rho.dy, rho.dy, 1)
cormat <- matrix(cormat.values, nrow = (nf + 1), ncol = (nf + 1))

# sample from a multivariate normal with specified correlation structure
ymean <- Dmean <- 2
mu <- as.matrix(c(Dmean, Dmean, Dmean, ymean))
eidecomp <- eigen(cormat, symmetric = TRUE)
L.sqrt <- diag(eidecomp$values^0.5)
cormat.sqrt <- eidecomp$vectors %>% L.sqrt %>% t(eidecomp$vectors)
Z <- matrix(rnorm(mydim[4] * (nf + 1)), nrow = mydim[4], ncol = (nf + 1))
Xw <- rep(1, mydim[4]) %>% t(mu) + Z %>% cormat.sqrt
Dmat <- Xw[, 1:nf]

# create a random four-way data tensor with D weights related to a response
Bmat <- matrix(runif(mydim[2] * nf), nrow = mydim[2], ncol = nf)
Cmat <- matrix(runif(mydim[3] * nf), nrow = mydim[3], ncol = nf)
nDd <- rep(c(10, 12, 14), length.out = mydim[4])
Gmat <- matrix(rnorm(nf * nf), nrow = nf)
Amat <- vector("list", mydim[4])
X <- Xmat <- Emat <- Amat
for (Dd in 1:mydim[4]) {
  Amat[[Dd]] <- matrix(nf * rnorm(nDd[Dd]), nrow = nDd[Dd], ncol = nf)
  Amat[[Dd]] <- svd(Amat[[Dd]], nv = 0)$u %>% Gmat
  leftMat <- Amat[[Dd]] %>% diag(Dmat[Dd,])
  Xmat[[Dd]] <- array(tcrossprod(leftMat, krprod(Cmat, Bmat)),
                    dim = c(nDd[Dd], mydim[2], mydim[3]))
  Emat[[Dd]] <- array(rnorm(nDd[Dd] * mydim[2] * mydim[3]),
                    dim = c(nDd[Dd], mydim[2], mydim[3]))
}

```

```

    X[[Dd]] <- Xmat[[Dd]] + Emat[[Dd]]
  }

# create a multiclass response
stor <- matrix(rep(1, nrow(Xw)), nrow = nrow(Xw))
stor[which(Xw[, (nf + 1)] < (ymean - 0.4 * sd(Xw[, (nf + 1)])))] <- 2
stor[which(Xw[, (nf + 1)] > (ymean + 0.4 * sd(Xw[, (nf + 1)])))] <- 0
y <- factor(stor)

# initialize
alpha <- seq(0, 1, length = 2)
gamma <- c(0, 1)
cost <- c(0.1, 5)
ntree <- c(200, 300)
nodesize <- c(1, 2)
size <- c(1, 2)
decay <- c(0, 1)
rda.alpha <- seq(0.1, 0.9, length = 2)
delta <- c(0.1, 1)
method <- c("PLR", "SVM", "RF", "NN", "RDA")
family <- "multinomial"
parameters <- list(alpha = alpha, gamma = gamma, cost = cost, ntree = ntree,
                  nodesize = nodesize, size = size, decay = decay,
                  rda.alpha = rda.alpha, delta = delta)

model <- "parafac2"
nfolds <- 3
nstart <- 5

# constrain first mode weights to be orthogonal, fourth mode to be nonnegative
const <- c("orthog", "uncons", "uncons", "nonneg")

# fit Parafac2 model and use fourth mode to tune classification methods
tune.object <- tune.cdfa(x = X, y = y, model = model, nfac = nf,
                      nfolds = nfolds, method = method, family = family,
                      parameters = parameters, parallel = FALSE,
                      const = const, nstart = nstart)

# create new data with Parafac2 structure and D weights related to response
mydim.new <- c(10, 11, 12, 10)
Znew <- matrix(rnorm(mydim.new[4] * (nf + 1)), nrow = mydim.new[4],
              ncol = (nf + 1))
Xwnew <- rep(1, mydim.new[4]) %>% t(mu) + Znew %>% cormat.sqrt
Dmatnew <- Xwnew[, 1:nf]
Amat <- vector("list", mydim.new[4])
Xnew <- Xmat <- Emat <- Amat
for (Dd in 1:mydim.new[4]) {
  Amat[[Dd]] <- matrix(nf * rnorm(nDd[Dd]), nrow = nDd[Dd], ncol = nf)
  Amat[[Dd]] <- svd(Amat[[Dd]), nv = 0)$u %>% Gmat
  leftMat <- Amat[[Dd]] %>% diag(Dmatnew[Dd, ])
  Xmat[[Dd]] <- array(tcrossprod(leftMat, krprod(Cmat, Bmat)),
                    dim = c(nDd[Dd], mydim.new[2], mydim.new[3]))
  Emat[[Dd]] <- array(rnorm(nDd[Dd] * mydim.new[2] * mydim.new[3]),
                    dim = c(nDd[Dd], mydim.new[2], mydim.new[3]))
}

```

```

  Xnew[[Dd]] <- Xmat[[Dd]] + Emat[[Dd]]
}

# create new random class labels for two levels
stor <- matrix(rep(1, nrow(Xwnew)), nrow = nrow(Xwnew))
stor[which(Xwnew[, (nf + 1)] < (ymean - 0.4 * sd(Xwnew[, (nf + 1)])))] <- 2
stor[which(Xwnew[, (nf + 1)] > (ymean + 0.4 * sd(Xwnew[, (nf + 1)])))] <- 0
newlabels <- as.numeric(stor)

# predict class labels
predict.labels <- predict(object = tune.object, newdata = Xnew,
                          type = "response")

# print predicted labels
predict.labels

```

print.cdfa

Print Method for Classification with Parallel Factor Analysis

Description

Prints summary of results from a 'cpfa' model object obtained using function `tune.cdfa`.

Usage

```
## S3 method for class 'cpfa'
print(x, ...)
```

Arguments

`x` A fit object of class 'cpfa' from function `tune.cdfa`.
`...` Additional print arguments.

Details

Prints names of the models and methods used to create the input 'cpfa' model object. Prints misclassification error rates and estimation times in seconds.

Value

Returns a summary of the 'cpfa' model object.

Author(s)

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References

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- Friedman, J. Hastie, T., and Tibshirani, R. (2010). Regularization paths for generalized linear models via coordinate descent. *Journal of Statistical Software*, 33(1), 1-22.
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- Helwig, N. (2017). Estimating latent trends in multivariate longitudinal data via Parafac2 with functional and structural constraints. *Biometrical Journal*, 59(4), 783-803.
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- Meyer, D., Dimitriadou, E., Hornik, K., Weingessel, A., and Leisch, F. (2023). e1071: Misc functions of the Department of Statistics, Probability Theory Group (Formerly: E1071), TU Wien. R Package Version 1.7-13.
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- Zou, H. and Hastie, T. (2005). Regularization and variable selection via the elastic net. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 67(2), 301-320.

Examples

```
##### Parafac example with 3-way array and binary response #####

# set seed and specify dimensions of a three-way tensor
set.seed(3)
mydim <- c(10, 11, 80)
nf <- 3

# create correlation matrix between response and third mode's weights
rho.cc <- .35
rho.cy <- .75
```

```

cormat.values <- c(1, rho.cc, rho.cc, rho.cy, rho.cc, 1, rho.cc, rho.cy,
                  rho.cc, rho.cc, 1, rho.cy, rho.cy, rho.cy, rho.cy, 1)
cormat <- matrix(cormat.values, nrow = (nf + 1), ncol = (nf + 1))

# sample from a multivariate normal with specified correlation structure
ymean <- Cmean <- 2
mu <- as.matrix(c(Cmean, Cmean, Cmean, ymean))
eidecomp <- eigen(cormat, symmetric = TRUE)
L.sqrt <- diag(eidecomp$values^0.5)
cormat.sqrt <- eidecomp$vectors %*% L.sqrt %*% t(eidecomp$vectors)
Z <- matrix(rnorm(mydim[3] * (nf + 1)), nrow = mydim[3], ncol = (nf + 1))
Xw <- rep(1, mydim[3]) %*% t(mu) + Z %*% cormat.sqrt
Cmat <- Xw[, 1:nf]

# create a random three-way data tensor with C weights related to a response
Amat <- matrix(rnorm(mydim[1] * nf), nrow = mydim[1], ncol = nf)
Bmat <- matrix(runif(mydim[2] * nf), nrow = mydim[2], ncol = nf)
Xmat <- tcrossprod(Amat, krprod(Cmat, Bmat))
Xmat <- array(Xmat, dim = mydim)
Emat <- array(rnorm(prod(mydim)), dim = mydim)
Emat <- nscale(Emat, 0, ssnew = sumsq(Xmat))
X <- Xmat + Emat

# create a binary response by dichotomizing at the specified response mean
y <- factor(as.numeric(Xw[, (nf + 1)] > ymean))

# initialize
alpha <- seq(0, 1, length = 2)
gamma <- c(0, 0.01)
cost <- c(1, 2)
ntree <- c(100, 200)
nodesize <- c(1, 2)
size <- c(1, 2)
decay <- c(0, 1)
rda.alpha <- c(0.1, 0.6)
delta <- c(0.1, 2)
method <- c("PLR", "SVM", "RF", "NN", "RDA")
family <- "binomial"
parameters <- list(alpha = alpha, gamma = gamma, cost = cost, ntree = ntree,
                  nodesize = nodesize, size = size, decay = decay,
                  rda.alpha = rda.alpha, delta = delta)

model <- "parafac"
nfolds <- 3

# constrain first mode weights to be orthogonal
const <- c("orthog", "uncons", "uncons")

# fit Parafac models and use third mode to tune classification methods
tune.object <- tune.cpfa(x = X, y = y, model = model, nfac = nf,
                      nfolds = nfolds, method = method, family = family,
                      parameters = parameters, parallel = FALSE,
                      const = const)

```



```
# print summary of output
print(tune.object)
```

tune.cpfa

Tuning for Classification with Parallel Factor Analysis

Description

Fits Richard A. Harshman's Parallel Factor Analysis-1 (Parafac) model or Parallel Factor Analysis-2 (Parafac2) model to a three-way or four-way data array. Allows for multiple constraint options on tensor modes. Uses component weights from a single mode of the model as predictors to tune parameters for one or more classification methods via a k-fold cross-validation procedure. Supports binary and multiclass classification.

Usage

```
tune.cpfa(x, y, model = c("parafac", "parafac2"), nfac = 1, nfolds = 10,
          method = c("PLR", "SVM", "RF", "NN", "RDA"),
          family = c("binomial", "multinomial"), parameters = list(),
          foldid = NULL, prior = NULL, cmode = NULL, parallel = FALSE,
          cl = NULL, verbose = TRUE, ...)
```

Arguments

x	Three-way or four-way data array. Must contain real numbers. See note below.
y	A vector containing at least two unique class labels. Should be a factor that contains two or more levels. For binary case, ensure the order of factor levels (left to right) is such that negative class is first and positive class is second.
model	Character designating the Parafac model to use, either <code>model = "parafac"</code> to fit the Parafac model or <code>model = "parafac2"</code> to fit the Parafac2 model.
nfac	Number of components for each Parafac or Parafac2 model to fit. Default is <code>nfac = 1</code> .
nfolds	Numeric setting number of folds for k-fold cross-validation. Must be 2 or greater. Default is <code>nfolds = 10</code> .
method	Character vector indicating classification methods to use. Possible methods include penalized logistic regression (PLR); support vector machine (SVM); random forest (RF); feed-forward neural network (NN); and regularized discriminant analysis (RDA). If none selected, default is to use all methods.
family	Character value specifying binary classification (<code>family = "binomial"</code>) or multiclass classification (<code>family = "multinomial"</code>). If not provided, number of levels of input y is used, where two levels is binary, and where three or more levels is multiclass.
parameters	List containing arguments related to classification methods. When specified, must contain one or more of the following:

	<p>alpha Values for penalized logistic regression alpha parameter; default is <code>alpha = seq(0, 1, length = 6)</code>. Must be numeric and contain only real numbers between 0 and 1, inclusive.</p> <p>lambda Optional user-supplied lambda sequence for <code>cv.glmnet</code> for penalized logistic regression. Default is <code>NULL</code>.</p> <p>cost Values for support vector machine cost parameter; default is <code>cost = c(1, 2, 4, 8, 16, 32, 64)</code>. Must be numeric and contain only real numbers greater than or equal to zero.</p> <p>gamma Values for support vector machine gamma parameter; default is <code>gamma = c(0, 0.01, 0.1, 1, 10, 100, 1000)</code>. Must be numeric and greater than or equal to 0.</p> <p>ntree Values for random forest number of trees parameter; default is <code>ntree = c(100, 200, 400, 600, 800, 1600, 3200)</code>. Must be numeric and contain only integers greater than or equal to 1.</p> <p>nodesize Values for random forest node size parameter; default is <code>nodesize = c(1, 2, 4, 8, 16, 32, 64)</code>. Must be numeric and contain only integers greater than or equal to 1.</p> <p>size Values for neural network size parameter; default is <code>size = c(1, 2, 4, 8, 16, 32, 64)</code>. Must be numeric and contain only integers greater than or equal to 0.</p> <p>decay Values for neural network decay parameter; default is <code>decay = c(0.001, 0.01, 0.1, 1, 2, 4, 8, 16)</code>. Must be numeric and contain only real numbers.</p> <p>rda.alpha Values for regularized discriminant analysis alpha parameter; default is <code>rda.alpha = seq(0, 0.999, length = 6)</code>. Must be numeric and contain only real numbers between 0 (inclusive) and 1 (exclusive).</p> <p>delta Values for regularized discriminant analysis delta parameter; default is <code>delta = c(0, 0.1, 1, 2, 3, 4)</code>. Must be numeric and contain only real numbers greater than or equal to 0.</p>
foldid	Vector containing fold IDs for k-fold cross-validation. Can be of class integer, numeric, or data frame. Should contain integers from 1 through the number of folds. If not provided, fold IDs are generated randomly for observations using 1 through the number of folds <code>nfolds</code> .
prior	Prior probabilities of class membership. If unspecified, the class proportions for input <code>y</code> are used. If specified, the probabilities should be in the order of the factor levels of input <code>y</code> .
cmode	Integer value of 1, 2, or 3 (or 4 if <code>x</code> is a four-way array) specifying the mode whose component weights will be predictors for classification. Defaults to the last mode of the inputted array (i.e., defaults to 3 for three-way array, and to 4 for four-way array). If <code>model = "parafac2"</code> , last mode will be used.
parallel	Logical indicating if parallel computing should be implemented. If <code>TRUE</code> , the package parallel is used for parallel computing. For all classification methods except penalized logistic regression, the doParallel package is used as a wrapper. Defaults to <code>FALSE</code> , which implements sequential computing.
c1	Cluster for parallel computing, which is used when <code>parallel = TRUE</code> . Note that if <code>parallel = TRUE</code> and <code>c1 = NULL</code> , then the cluster is defined as <code>makeCluster(detectCores())</code> .

verbose	If TRUE, progress is printed.
...	Additional arguments to be passed to function <code>parafac</code> for fitting a Parafac model or function <code>parafac2</code> for fitting a Parafac2 model. Example: can impose different constraints on different modes of the input array using the argument <code>const</code> . See help file for function <code>parafac</code> or for function <code>parafac2</code> for additional details.

Details

After fitting a Parafac or Parafac2 model with package **multiway** (see `parafac` or `parafac2` in **multiway** for details), the estimated classification mode weight matrix is passed to one or several of five classification methods—including penalized logistic regression (PLR); support vector machine (SVM); random forest (RF); feed-forward neural network (NN); and regularized discriminant analysis (RDA).

Package **glmnet** fits models for PLR. PLR tunes penalty parameter λ while the elastic net parameter α is set by the user (see the help file for function `cv.glmnet` in package **glmnet**). For SVM, package **e1071** is used with a radial basis kernel. Penalty parameter cost and radial basis parameter γ are used (see `svm` in package **e1071**). For RF, package **randomForest** is used and implements Breiman's random forest algorithm. The number of predictors sampled at each node split is set at the default of \sqrt{R} , where R is the number of Parafac or Parafac2 components. Two tuning parameters allowed are `ntree`, the number of trees to be grown, and `nodesize`, the minimum size of terminal nodes (see `randomForest` in package **randomForest**). For NN, package **nnet** fits a single-hidden-layer, feed-forward neural network model. Penalty parameters `size` (i.e., number of hidden layer units) and `decay` (i.e., weight decay) are used (see **nnet**). For RDA, package **rda** fits a shrunken centroids regularized discriminant analysis model. Tuning parameters include `rda.alpha`, the shrinkage penalty for the within-class covariance matrix, and `delta`, the shrinkage penalty of class centroids towards the overall dataset centroid.

For all five methods, k -fold cross-validation is implemented to tune classification parameters where the number of folds is set by argument `nfolds`.

Value

Returns an object of class `cpfa` with the following elements:

<code>opt.model</code>	List containing optimal model for tuned classification methods for each Parafac or Parafac2 model that was fit.
<code>opt.param</code>	Data frame containing optimal parameters for tuned classification methods.
<code>kcv.error</code>	Data frame containing KCV misclassification error for optimal parameters for tuned classification methods.
<code>est.time</code>	Data frame containing times for fitting Parafac or Parafac2 model and for tuning classification methods.
<code>method</code>	Numeric indicating classification methods used. Value of '1' indicates 'PLR'; value of '2' indicates 'SVM'; value of '3' indicates 'RF'; value of '4' indicates 'NN'; and value of '5' indicates 'RDA'.
<code>x</code>	Three-way or four-way array used. If a list was used with <code>model = "parafac2"</code> , returns list of matrices or three-way arrays used.

<code>y</code>	Factor containing class labels used. Note that output <code>y</code> is recoded such that the input labels of <code>y</code> are converted to numeric integers from 0 through the number of levels, which are then applied as labels for output <code>y</code> .
<code>Aweights</code>	List containing estimated A weights for each Parafac or Parafac2 model that was fit.
<code>Bweights</code>	List containing estimated B weights for each Parafac or Parafac2 model that was fit.
<code>Cweights</code>	List containing estimated C weights for each Parafac or Parafac2 model that was fit. Null if inputted argument <code>x</code> was a three-way array.
<code>Phi</code>	If <code>model = "parafac2"</code> , a list containing estimated Phi from the Parafac2 model. Phi is the common cross product matrix shared by all levels of the last mode (see help file for function <code>parafac2</code> in package multiway for additional details). NULL if <code>model = "parafac"</code> .
<code>const</code>	Constraints used in fitting Parafac or Parafac2 models. If argument <code>const</code> was not inputted, no constraints will be used.
<code>cmode</code>	Integer value of 1, 2, or 3 (or 4 if <code>x</code> is a four-way array) specifying mode whose component weights were predictors for classification.
<code>family</code>	Character value specifying whether classification was binary (<code>family = "binomial"</code>) or multiclass (<code>family = "multinomial"</code>).
<code>xdim</code>	Numeric value specifying number of levels for each mode of input <code>x</code> . If <code>model = "parafac2"</code> , number of levels for first mode is designated as NA because the number of levels can differ across levels of the last mode.
<code>lxdim</code>	Numeric value specifying number of modes of input <code>x</code> .
<code>train.weights</code>	List containing classification component weights for each fit Parafac or Parafac2 model, for possibly different numbers of components. The weights used to train classifiers.

Note

For fitting the Parafac model, if argument `cmode` is not null, input array `x` is reshaped with function `aperm` such that the `cmode` dimension of `x` is ordered last. Estimated mode A and B (and mode C for a four-way array) weights that are outputted as `Aweights` and `Bweights` (and `Cweights`) reflect this permutation. For example, if `x` is a four-way array and `cmode = 2`, the original input modes 1, 2, 3, and 4 will correspond to output modes 1, 3, 4, 2. Here, output A = input 1; B = 3, and C = 4 (i.e., the second mode specified by `cmode` has been moved to the D mode/last mode). For `model = "parafac2"`, classification mode is assumed to be the last mode (i.e., mode C for three-way array and mode D for four-way array).

In addition, note that the following combination of arguments will give an error: `nfac = 1`, `family = "multinomial"`, `method = "PLR"`. The issue arises from providing `glmnet::cv.glmnet` input `x` a matrix with a single column. The issue is resolved for `family = "binomial"` because a column of 0s is appended to the single column, but this solution does not appear to work for the multiclass case. As such, this combination of arguments is not currently allowed. This issue will be resolved in a future update.

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Examples

```
##### Parafac example with 3-way array and binary response #####

# set seed and specify dimensions of a three-way tensor
set.seed(3)
mydim <- c(10, 11, 80)
nf <- 3

# create correlation matrix between response and third mode's weights
rho.cc <- .35
rho.cy <- .75
```

```

cormat.values <- c(1, rho.cc, rho.cc, rho.cy, rho.cc, 1, rho.cc, rho.cy,
                  rho.cc, rho.cc, 1, rho.cy, rho.cy, rho.cy, rho.cy, 1)
cormat <- matrix(cormat.values, nrow = (nf + 1), ncol = (nf + 1))

# sample from a multivariate normal with specified correlation structure
ymean <- Cmean <- 2
mu <- as.matrix(c(Cmean, Cmean, Cmean, ymean))
eidecomp <- eigen(cormat, symmetric = TRUE)
L.sqrt <- diag(eidecomp$values^0.5)
cormat.sqrt <- eidecomp$vectors %*% L.sqrt %*% t(eidecomp$vectors)
Z <- matrix(rnorm(mydim[3] * (nf + 1)), nrow = mydim[3], ncol = (nf + 1))
Xw <- rep(1, mydim[3]) %*% t(mu) + Z %*% cormat.sqrt
Cmat <- Xw[, 1:nf]

# create a random three-way data tensor with C weights related to a response
Amat <- matrix(rnorm(mydim[1] * nf), nrow = mydim[1], ncol = nf)
Bmat <- matrix(runif(mydim[2] * nf), nrow = mydim[2], ncol = nf)
Xmat <- tcrossprod(Amat, krprod(Cmat, Bmat))
Xmat <- array(Xmat, dim = mydim)
Emat <- array(rnorm(prod(mydim)), dim = mydim)
Emat <- nscale(Emat, 0, ssnew = sumsq(Xmat))
X <- Xmat + Emat

# create a binary response by dichotomizing at the specified response mean
y <- factor(as.numeric(Xw[, (nf + 1)] > ymean))

# initialize
alpha <- seq(0, 1, length = 2)
gamma <- c(0, 0.01)
cost <- c(1, 2)
ntree <- c(100, 200)
nodesize <- c(1, 2)
size <- c(1, 2)
decay <- c(0, 1)
rda.alpha <- c(0.1, 0.6)
delta <- c(0.1, 2)
method <- c("PLR", "SVM", "RF", "NN", "RDA")
family <- "binomial"
parameters <- list(alpha = alpha, gamma = gamma, cost = cost, ntree = ntree,
                  nodesize = nodesize, size = size, decay = decay,
                  rda.alpha = rda.alpha, delta = delta)

model <- "parafac"
nfolds <- 3

# constrain first mode weights to be orthogonal
const <- c("orthog", "uncons", "uncons")

# fit Parafac models and use third mode to tune classification methods
tune.object <- tune.cpfa(x = X, y = y, model = model, nfac = nf,
                      nfolds = nfolds, method = method, family = family,
                      parameters = parameters, parallel = FALSE,
                      const = const)

```

```

# print tuning object
tune.object

##### Parafac2 example with 4-way array and multiclass response #####

# set seed and specify dimensions of a four-way tensor
set.seed(5)
mydim <- c(10, 11, 12, 90)
nf <- 3

# create correlation matrix between response and fourth mode's weights
rho.dd <- .35
rho.dy <- .75
cormat.values <- c(1, rho.dd, rho.dd, rho.dy, rho.dd, 1, rho.dd, rho.dy,
                  rho.dd, rho.dd, 1, rho.dy, rho.dy, rho.dy, rho.dy, 1)
cormat <- matrix(cormat.values, nrow = (nf + 1), ncol = (nf + 1))

# sample from a multivariate normal with specified correlation structure
ymean <- Dmean <- 2
mu <- as.matrix(c(Dmean, Dmean, Dmean, ymean))
eidecomp <- eigen(cormat, symmetric = TRUE)
L.sqrt <- diag(eidecomp$values^0.5)
cormat.sqrt <- eidecomp$vectors %>% L.sqrt %>% t(eidecomp$vectors)
Z <- matrix(rnorm(mydim[4] * (nf + 1)), nrow = mydim[4], ncol = (nf + 1))
Xw <- rep(1, mydim[4]) %>% t(mu) + Z %>% cormat.sqrt
Dmat <- Xw[, 1:nf]

# create a random four-way data tensor with D weights related to a response
Bmat <- matrix(runif(mydim[2] * nf), nrow = mydim[2], ncol = nf)
Cmat <- matrix(runif(mydim[3] * nf), nrow = mydim[3], ncol = nf)
nDd <- rep(c(10, 12, 14), length.out = mydim[4])
Gmat <- matrix(rnorm(nf * nf), nrow = nf)
Amat <- vector("list", mydim[4])
X <- Xmat <- Emat <- Amat
for (Dd in 1:mydim[4]) {
  Amat[[Dd]] <- matrix(nf * rnorm(nDd[Dd]), nrow = nDd[Dd], ncol = nf)
  Amat[[Dd]] <- svd(Amat[[Dd]], nv = 0)$u %>% Gmat
  leftMat <- Amat[[Dd]] %>% diag(Dmat[Dd,])
  Xmat[[Dd]] <- array(tcrossprod(leftMat, krprod(Cmat, Bmat)),
                    dim = c(nDd[Dd], mydim[2], mydim[3]))
  Emat[[Dd]] <- array(rnorm(nDd[Dd] * mydim[2] * mydim[3]),
                    dim = c(nDd[Dd], mydim[2], mydim[3]))
  X[[Dd]] <- Xmat[[Dd]] + Emat[[Dd]]
}

# create a multiclass response
stor <- matrix(rep(1, nrow(Xw)), nrow = nrow(Xw))
stor[which(Xw[, (nf + 1)] < (ymean - 0.4 * sd(Xw[, (nf + 1)]))]] <- 2
stor[which(Xw[, (nf + 1)] > (ymean + 0.4 * sd(Xw[, (nf + 1)]))]] <- 0
y <- factor(stor)

# initialize
alpha <- seq(0, 1, length = 2)

```

```
gamma <- c(0, 1)
cost <- c(0.1, 5)
ntree <- c(200, 300)
nodesize <- c(1, 2)
size <- c(1, 2)
decay <- c(0, 1)
rda.alpha <- seq(0.1, 0.9, length = 2)
delta <- c(0.1, 1)
method <- c("PLR", "SVM", "RF", "NN", "RDA")
family <- "multinomial"
parameters <- list(alpha = alpha, gamma = gamma, cost = cost, ntree = ntree,
                  nodesize = nodesize, size = size, decay = decay,
                  rda.alpha = rda.alpha, delta = delta)
model <- "parafac2"
nfolds <- 3
nstart <- 5

# constrain first mode weights to be orthogonal, fourth mode to be nonnegative
const <- c("orthog", "uncons", "uncons", "nonneg")

# fit Parafac2 model and use fourth mode to tune classification methods
tune.object <- tune.cpfa(x = X, y = y, model = model, nfac = nf,
                      nfolds = nfolds, method = method, family = family,
                      parameters = parameters, parallel = FALSE,
                      const = const, nstart = nstart)

# print summary of output
print(tune.object)
```


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