

# Package ‘equateIRT’

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**Title** IRT Equating Methods

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**Description** Computation of direct, chain and average (bisector) equating coefficients with standard errors using Item Response Theory (IRT) methods for dichotomous items (Battauz (2013) <[doi:10.1007/s11336-012-9316-y](https://doi.org/10.1007/s11336-012-9316-y)>, Battauz (2015) <[doi:10.18637/jss.v068.i07](https://doi.org/10.18637/jss.v068.i07)>). Test scoring can be performed by true score equating and observed score equating methods. DIF detection can be performed using a Wald-type test (Battauz (2018) <[doi:10.1007/s10260-018-00442-w](https://doi.org/10.1007/s10260-018-00442-w)>).

**License** GPL-3

**VignetteBuilder** knitr

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equateIRT-package	<i>IRT Equating Methods</i>
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## Description

This package computes direct, chain and average (bisector) equating coefficients with standard errors using IRT methods for dichotomous items. The IRT models included are the three-parameter logistic model, the two-parameter logistic model, the one-parameter logistic model and the Rasch model. Test scoring can be performed by true score equating and observed score equating methods. DIF detection can be performed using a Wald-type test.

## Details

Package: equateIRT  
 Type: Package  
 Version: 2.2.1  
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Direct equating coefficients and their standard errors between forms presenting common items can be computed using function `direc`. The equating methods implemented are "mean-mean", "mean-geometric mean", "mean-sigma", "Haebara" and "Stocking-Lord". Estimates of item parameters and their covariance matrix can be imported from the R packages `ltm` and `mirt` or from the IRT programs `IRTPRO` and `flexMIRT` using functions `import.ltm`, `import.mirt`, `import.irtpro` and `import.flexmirt`. Item parameter estimates from other software can be imported as well by the user. Data should be previously organized using function `modIRT`. Function `alldirec` computes all direct equating coefficients (with standard errors) between all pairs of a list of forms. Function `chainec` computes chain equating coefficients (and standard errors) given direct equating coeffi-

coefficients between forms directly linked. Average equating coefficients with standard errors can be calculated using function `bisectorec`, that implements the (weighted) bisector method. Once the equating coefficients are obtained, the computation of equated scores can be performed using function `score`, which implements true score equating and observed score equating. Standard errors of equated scores are also provided. Three simulated datasets are available for illustrative purposes. These datasets contain item parameter coefficients and their covariance matrix. In particular, `est3pl` concerns a three-parameter logistic model, `est2pl` regards a two-parameter logistic model, and `estrasch` refers to a Rasch model. The estimates included in `est2pl` are obtained from the dataset `data2pl`, also contained in the package. Function `dif.test` performs a Wald-type test for the detection of DIF (Battaaz, 2018).

### Author(s)

Michela Battaaz

Maintainer: Michela Battaaz <michela.battaaz@uniud.it>

### References

- Battaaz, M. (2013). IRT Test Equating in Complex Linkage Plans. *Psychometrika*, **78**, 464–480.
- Battaaz, M. (2015). equateIRT: An R Package for IRT Test Equating. *Journal of Statistical Software*, **68**, 1–22.
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- Chalmers, R. P. (2012). mirt: A Multidimensional Item Response Theory Package for the R Environment. *Journal of Statistical Software*, **48**, 1–29.
- Holland, P.W. and Strawderman, W.E. (2011). How to average equating functions if you must. In A.A. von Davier (Ed.), *Statistical models for test equating, scaling, and linking* (pp. 89–107). New York: Springer.
- Kolen, M.J. and Brennan, R.L. (2014). *Test equating, scaling, and linking: methods and practices*, 3rd ed., New York: Springer.
- Ogasawara, H. (2000). Asymptotic standard errors of IRT equating coefficients using moments. *Economic Review (Otaru University of Commerce)*, **51**, 1–23.
- Ogasawara, H. (2001a). Item response theory true score equatings and their standard errors. *Journal of Educational and Behavioral Statistics*, **26**, 31–50.
- Ogasawara, H. (2001b). Standard Errors of Item Response Theory Equating/Linking by Response Function Methods. *Applied Psychological Measurement*, **25**, 53–67.
- Ogasawara, H. (2003). Asymptotic standard errors of IRT observed-score equating methods. *Psychometrika*, **68**, 193–211.
- Rizopoulos, D. (2006). ltm: an R package for latent variable modelling and item response theory analyses. *Journal of Statistical Software*, **17**, 1–25.

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alldirec

*Direct Equating Coefficients Between All Pairs of a List of Forms*


---

### Description

Calculates direct equating coefficients and standard errors using IRT methods between all pairs of a list of forms.

### Usage

```
alldirec(mods, method = "mean-mean", all = FALSE, quadrature = TRUE, nq = 30,
direction = "both")
```

### Arguments

mods	an object of the class modIRT containing item parameter coefficients and their covariance matrix of the forms to be equated.
method	the equating method to be used. This should be one of "mean-mean", "mean-gmean", "mean-sigma", "Haebara" or "Stocking-Lord".
all	logical; if FALSE forms that do not have common items will not appear in the output.
quadrature	logical; if TRUE the Gauss-Hermite quadrature is used to approximate the integral in the function that is minimized in the Haebara and Stocking-Lord methods. If FALSE the integral is replaced with a sum over 40 equally spaced values ranging from -4 to 4 with an increment of 0.05 and weights equal to one for all values.
nq	number of quadrature points used for the Gauss-Hermite quadrature if quadrature is TRUE
direction	a character string specifying the direction used to equate pair of forms. If "back" newer forms are equated to older forms, if "forward" older forms are equated to newer forms, if "both" (the default) equating is performed in both directions. Form A is considered to be newer than Form B if it is listed after Form B in mods.

### Value

An object of class eqclist consisting in a list with length equal to the number of pairs of forms equated. Each component of the list is an object of class eqc returned by function [direc](#).

### Author(s)

Michela Battauz

## References

- Battaaz, M. (2015). equateIRT: An R Package for IRT Test Equating. *Journal of Statistical Software*, **68**, 1–22.
- Kolen, M.J. and Brennan, R.L. (2014). *Test equating, scaling, and linking: methods and practices*, 3rd ed., New York: Springer
- Ogasawara, H. (2000). Asymptotic standard errors of IRT equating coefficients using moments. *Economic Review (Otaru University of Commerce)*, **51**, 1–23.
- Ogasawara, H. (2001). Standard Errors of Item Response Theory Equating/Linking by Response Function Methods. *Applied Psychological Measurement*, **25**, 53–67.

## See Also

[direc](#), [eqc](#), [itm](#), [modIRT](#), [summary.eqclist](#)

## Examples

```
# three-parameter logistic model
# direct equating coefficients using the "Stocking-Lord" method
data(est3pl)
test <- paste("test", 1:5, sep = "")
mod3pl <- modIRT(coef = est3pl$coef, var = est3pl$var, names = test, display = FALSE)
direclist3pl <- alldirec(mods = mod3pl, method = "Stocking-Lord")
summary(direclist3pl)
summary(direclist3pl$test1.test2)

# two-parameter logistic model
# direct equating coefficients using the "Haebara" method
data(est2pl)
test <- paste("test", 1:5, sep = "")
mod2pl <- modIRT(coef = est2pl$coef, var = est2pl$var, names = test, display = FALSE)
direclist2pl <- alldirec(mods = mod2pl, method = "Haebara")
summary(direclist2pl)
summary(direclist2pl$test1.test5)

# Rasch model
# direct equating coefficients using the "mean-mean" method
data(estrasch)
test <- paste("test", 1:5, sep = "")
modrasch <- modIRT(coef = estrasch$coef, var = estrasch$var, names = test,
display = FALSE)
direclistrasch <- alldirec(mods = modrasch, method = "mean-mean", all = TRUE)
summary(direclistrasch)
summary(direclistrasch$test5.test4)
```

bisectorec

*Bisector Equating Coefficients***Description**

Calculates average equating coefficients using the bisector method and standard errors given a set of direct and chain equating coefficients.

**Usage**

```
bisectorec(ecall, mods = NULL, weighted = TRUE, unweighted = TRUE)
```

**Arguments**

ecall	list of objects of class eqc or ceqc returned by functions <a href="#">direc</a> and <a href="#">chainec</a> .
mods	an object of class modIRT containing item parameter coefficients and their covariance matrix of the forms to be equated. From version 2.0 it can be NULL
weighted	logical; if TRUE weighted bisector coefficients are computed.
unweighted	logical; if TRUE unweighted bisector coefficients are computed.

**Value**

An object of class meqc with components

coef	data frame containing link, path, coefficient A, coefficient B, standard error of coefficient A (seA), standard error of coefficient B (seB) and weights of direct, chain and bisector equating coefficients.
method	the equating method used.

**Author(s)**

Michela Battauz

**References**

Battauz, M. (2013). IRT Test Equating in Complex Linkage Plans. *Psychometrika*, **78**, 464–480.

Battauz, M. (2015). equateIRT: An R Package for IRT Test Equating. *Journal of Statistical Software*, **68**, 1–22.

Holland, P.W. and Strawderman, W.E. (2011). How to average equating functions if you must. In A.A. von Davier (Ed.), *Statistical models for test equating, scaling, and linking* (pp. 89–107). New York: Springer.

**See Also**

[chainec](#), [convert](#), [direc](#), [eqc](#), [summary.meqc](#)

**Examples**

```

# three-parameter logistic model
# direct equating coefficients using the "Stocking-Lord" method
data(est3pl)
test <- paste("test", 1:5, sep = "")
mod3pl <- modIRT(coef = est3pl$coef, var = est3pl$var, names = test, display = FALSE)
direclist3pl <- alldirec(mods = mod3pl, method = "Stocking-Lord")
# compute chain equating coefficients for path 1,2,3,4,5
pth3 <- paste("test", 1:5, sep = "")
chainec3 <- chainec(direclist = direclist3pl, pths = pth3)
# create a list of objects of class eqc or ceqc
ecall <- c(chainec3, direclist3pl["test1.test5"])
# compute bisector and weighted bisector coefficients
allec <- bisectorec(ecall = ecall, weighted = TRUE, unweighted = TRUE)
summary(allec)

# two-parameter logistic model
# direct equating coefficients using the "Haebara" method
data(est2pl)
test <- paste("test", 1:5, sep = "")
mod2pl <- modIRT(coef = est2pl$coef, var = est2pl$var, names = test, display = FALSE)
direclist2pl <- alldirec(mods = mod2pl, method = "Haebara")
# compute chain equating coefficients for path 1,2,3,4
pth1 <- paste("test", 1:4, sep = "")
chainec1 <- chainec(direclist = direclist2pl, pths = pth1)
# compute chain equating coefficients for path 1,5,4
pth2 <- c(paste("test", c(1,5,4), sep = ""))
chainec2 <- chainec(direclist = direclist2pl, pths = pth2)
# compute chain equating coefficients for path 1,2,3,4,5
pth3 <- paste("test", 1:5, sep = "")
chainec3 <- chainec(direclist = direclist2pl, pths = pth3)
# create a list of objects of class eqc or ceqc
ecall <- c(chainec1, chainec2, chainec3, direclist2pl["test1.test5"])
# compute bisector and weighted bisector coefficients
allec <- bisectorec(ecall = ecall, weighted = TRUE, unweighted = TRUE)
summary(allec)

# Rasch model
# direct equating coefficients using the "mean-mean" method
data(estrasch)
test <- paste("test", 1:5, sep = "")
modrasch <- modIRT(coef = estrasch$coef, var = estrasch$var, names = test,
display = FALSE)
direclistrasch <- alldirec(mods = modrasch, method = "mean-mean", all = TRUE)
# compute chain equating coefficients for path 1,2,3,4
pth1 <- paste("test", 1:4, sep = "")
chainec1 <- chainec(direclist = direclistrasch, pths = pth1)
# compute chain equating coefficients for path 1,5,4
pth2 <- c(paste("test", c(1,5,4), sep = ""))
chainec2 <- chainec(direclist = direclistrasch, pths = pth2)
# create a list of objects of class eqc or ceqc
ecall <- c(chainec1, chainec2)

```

```
# compute bisector and weighted bisector coefficients
altec <- bisectorec(ecall = ecall, weighted = TRUE, unweighted = TRUE)
summary(altec)
```

---

chainec

*Chain Equating Coefficients*


---

## Description

Calculates chain (indirect) equating coefficients and standard errors using IRT methods.

## Usage

```
chainec(r = NULL, direclist, f1 = NULL, f2 = NULL, pths = NULL)
```

## Arguments

r	length of the chain, that is the number of forms used for equating including extremes. It should be at least 3. It does not need to be specified if argument pths is not NULL.
direclist	an object of the class eqclist return by function <a href="#">alldirec</a> containing direct equating coefficients between pairs of forms.
f1	the name of the first form of the chain.
f2	the name of the last form of the chain.
pths	vector, matrix or data frame containing the path used for equating. The number of columns is equal to r and the number of rows is equal to the equatings that have to be performed. If NULL all the chain equating coefficients of length r will be computed.

## Details

Equating coefficients perform the conversion from the scale of the first form to the scale of the last form of the path.

## Value

An object of class ceqclist consisting in a list with length equal to the number of chain equating coefficients computed. Each component of the list is an object of class ceqc with components

tab1	item parameters of the first form.
tab2	item parameters of the last form.
tab	Data frame containing item names (Item), item parameters of the first form (e.g. test1), item parameters of the last form (e.g. test3), and item parameters of the first form converted in the scale of the last form (e.g. test1.as.test3).
varAll	covariance matrix of item parameters of all forms used in the chain.

partial	partial derivatives of equating coefficients A and B with respect to item parameters.
A	equating coefficient A.
B	equating coefficient B.
varAB	covariance matrix of the equating coefficients.
commonitem	list of length r-1 containing the names of common item parameters between adjacent forms.
ni	vector containing number of common items between pairs of adjacent forms.
forms	names of equated forms.
method	the equating method used.
itmp	number of item parameters of the IRT model.

**Author(s)**

Michela Battauz

**References**

- Battauz, M. (2013). IRT Test Equating in Complex Linkage Plans. *Psychometrika*, **78**, 464–480.
- Battauz, M. (2015). equateIRT: An R Package for IRT Test Equating. *Journal of Statistical Software*, **68**, 1–22.
- Kolen, M.J. and Brennan, R.L. (2014). *Test equating, scaling, and linking: methods and practices*, 3rd ed., New York: Springer

**See Also**

[alldirec](#), [eqc](#), [itm](#), [summary.ceqc](#), [summary.ceqclist](#)

**Examples**

```
# three-parameter logistic model
# direct equating coefficients using the "Stocking-Lord" method
data(est3pl)
test<-paste("test", 1:5, sep = "")
mod3pl <- modIRT(coef = est3pl$coef, var = est3pl$var, names = test, display = FALSE)
direclist3pl <- alldirec(mods = mod3pl, method = "Stocking-Lord")
# compute all chain equating coefficients of length 4
chainec4 <- chainec(r = 4, direclist = direclist3pl)
summary(chainec4)
summary(chainec4$test1.test2.test3.test4)
# compute all chain equating coefficients of length 4
# where the first form is test1
chainec4.1 <- chainec(r = 4, direclist = direclist3pl, f1 = "test1")
summary(chainec4.1)
# compute all chain equating coefficients of length 4
# where the first form is test1 and the last form is test4
chainec4.14 <- chainec(r = 4, direclist = direclist3pl, f1 = "test1", f2 = "test4")
summary(chainec4.14)
```

```

# two-parameter logistic model
# direct equating coefficients using the "Haebara" method
data(est2pl)
test<-paste("test", 1:5, sep = "")
mod2pl <- modIRT(coef = est2pl$coef, var = est2pl$var, names = test, display = FALSE)
direclist2pl <- alldirec(mods = mod2pl, method = "Haebara")
# compute chain equating coefficients of a given path
pth <- paste("test", c(1,5,4), sep = "")
chainec154 <- chainec(direclist = direclist2pl, pths = pth)
summary(chainec154)

```

```

# Rasch model
# direct equating coefficients using the "mean-mean" method
data(estrasch)
test<-paste("test", 1:5, sep = "")
modrasch <- modIRT(coef = estrasch$coef, var = estrasch$var, names = test,
display = FALSE)
direclistrasch <- alldirec(mods = modrasch, method = "mean-mean", all = TRUE)
# compute chain equating coefficients of two given paths
pth1 <- paste("test", 1:3, sep = "")
pth2 <- paste("test", c(1,5,4), sep = "")
pths <- rbind(pth1, pth2)
chainec1 <- chainec(direclist = direclistrasch, pths = pths)
summary(chainec1)

```

---

convert

*Item Parameters Conversion*

---

## Description

Converts item and person parameters using equating coefficients.

## Usage

```
convert(A, B, coef = NULL, person.par = NULL)
```

## Arguments

A	equating coefficient A.
B	equating coefficient B.
coef	vector of item parameters return by function <a href="#">modIRT</a> .
person.par	vector of person parameters estimates.

**Details**

Difficulty parameters  $b$  are converted using transformation  $b \cdot A + B$ ; discrimination parameters  $a$  are converted using transformation  $a/A$ ; guessing parameters  $c$  are not transformed.

Person parameters  $\theta$  are converted using transformation  $\theta \cdot A + B$ .

**Value**

A list with components

coef                vector of item parameters transformed.  
 person.par        vector of person parameters transformed

**Author(s)**

Michela Battauz

**References**

Battauz, M. (2015). equateIRT: An R Package for IRT Test Equating. *Journal of Statistical Software*, **68**, 1–22.

Kolen, M.J. and Brennan, R.L. (2014). *Test equating, scaling, and linking: methods and practices*, 3rd ed., New York: Springer

**See Also**

[itm](#), [modIRT](#)

**Examples**

```
# conversion using direct coefficients
# three-parameter logistic model
# direct equating coefficients between forms 1 and 2 using the Stocking-Lord method
data(est3pl)
test <- paste("test", 1:5, sep = "")
mod3pl <- modIRT(coef = est3pl$coef, var = est3pl$var, names = test, display = FALSE)
l12 <- direc(mod1 = mod3pl[1], mod2 = mod3pl[2], method = "Stocking-Lord")
convert(A = l12$A, B = l12$B, coef = coef(mod3pl$test1))
# the conversion of item parameters is obtained also using
itm(l12)

# conversion using bisector coefficients
# two-parameter logistic model
# direct equating coefficients using the "Haebara" method
data(est2pl)
test <- paste("test", 1:5, sep = "")
mod2pl <- modIRT(coef = est2pl$coef, var = est2pl$var, names = test, display = FALSE)
direclist2pl <- alldirec(mods = mod2pl, method = "Haebara")
# compute chain equating coefficients for path 1,2,3,4
pth1 <- paste("test", 1:4, sep = "")
chainec1 <- chainec(direclist = direclist2pl, pths = pth1)
```

```
# compute chain equating coefficients for path 1,5,4
pth2 <- c(paste("test", c(1,5,4), sep = ""))
chainec2 <- chainec(direclist = direclist2pl, pths = pth2)
# create a list of objects of class ceqc
ecall <- c(chainec1, chainec2)
# compute bisector and weighted bisector coefficients
allec <- bisectorec(ecall = ecall, weighted = TRUE, unweighted = TRUE)
summary(allec)
eqc14<-eqc(allec, link = "test1.test4", path = "bisector")
convert(A = eqc14$A, B = eqc14$B, coef = coef(mod2pl$test1), person.par = seq(-3, 3, 0.5))
```

---

data2pl

*Simulated Data Sets*

---

### **Description**

Five simulated data sets from a two-parameter logistic model.

### **Usage**

```
data(data2pl)
```

### **Format**

A list of length 5, containing 5 data frames with 5000 dichotomous responses to 20 items.

### **Author(s)**

Michela Battauz

### **See Also**

[est2pl](#), [import.ltm](#)

### **Examples**

```
data(data2pl)
data2pl[[1]][1:3,]
```

---

`dataDIF`*Simulated Data Set with DIF*

---

**Description**

Item responses and group membership.

**Usage**

```
data(dataDIF)
```

**Format**

A dataframe with 3000 dichotomous responses to 20 items (I01 to I20) and group membership (group).

**Author(s)**

Michela Battauz

**See Also**

[dif.test](#)

**Examples**

```
data(dataDIF)
```

---

`dif.test`*DIF Test*

---

**Description**

Performs a Wald-type test for Differential Item Functioning detection.

**Usage**

```
dif.test(coef, var, names = NULL, reference = NULL, method = "mean-mean",  
         quadrature = TRUE, nq = 30, DIFtype = NULL, purification = FALSE,  
         signif.level = 0.05, trace = FALSE, maxiter = 30, anchor = NULL)
```

### Arguments

coef	list of matrices (one for each group) containing the item parameter estimates. Guessing, difficulty and discrimination parameters should strictly be given in this order and they are contained in different columns of the matrix. The names of the rows of each matrix should be the names of the items.
var	list of matrices (one for each group) containing the covariance matrix of item parameter estimates. They should be given in the same order of coefficients.
names	character vector containing the names of the groups. This should have the same length of coef and var. If NULL, the names of the groups will be "T1", "T2", ...
reference	reference group. Can be specified by name or number. The default is the first group.
method	the equating method to be used in function <code>direc</code> to convert the item parameters to the scale of the reference group. This should be one of "mean-mean", "mean-gmean", "mean-sigma", "Haebara" or "Stocking-Lord".
quadrature	logical; if TRUE the Gauss-Hermite quadrature is used in function <code>direc</code> to approximate the integral in the function that is minimized in the Haebara and Stocking-Lord methods. If FALSE the integral is replaced with a sum over 40 equally spaced values ranging from -4 to 4 with an increment of 0.05 and weights equal to one for all values.
nq	number of quadrature points used for the Gauss-Hermite quadrature if quadrature is TRUE.
DIFtype	character indicating which parameters to test for DIF. If NULL all parameters are tested for DIF. Use "beta1" for $\beta_1$ , "beta2" for $\beta_2$ , "beta3" for $\beta_3$ , "beta12" for $\beta_1$ and $\beta_2$ , "beta123" for $\beta_1$ , $\beta_2$ and $\beta_3$ . See details.
purification	logical. if TRUE the procedure described in Candell and Drasgow (1988) is applied.
signif.level	significance level to use in the purification process.
trace	logical. If TRUE tracing information is produced.
maxiter	The maximum number of iterations in the purification process.
anchor	Optional character vector containing the names of the items to use for equating. These should be items free of DIF.

### Details

The parameterization of the IRT model is that commonly used for estimation. Under this parameterization, the three-parameter logistic model is as follows

$$\pi_i = c_i + (1 - c_i) \frac{\exp(\beta_{1i} + \beta_{2i}z)}{1 + \exp(\beta_{1i} + \beta_{2i}z)},$$

where  $\pi_i$  denotes the conditional probability of responding correctly to the  $i$ th item given  $z$ ,  $c_i$  denotes the guessing parameter,  $\beta_{1i}$  is the easiness parameter,  $\beta_{2i}$  is the discrimination parameter, and  $z$  denotes the latent ability. Furthermore, the guessing parameter is equal to

$$c_i = \frac{\exp(\beta_{3i})}{1 + \exp(\beta_{3i})}$$

The test verifies whether the item parameters  $\beta_{1i}, \beta_{2i}, \beta_{3i}$  are invariant across two or more groups as explained in Battauz (2018).

### Value

An object of class `dif.t` with components

<code>test</code>	matrix containing the test statistic and the p-value for each item. "noGuess" is equal to 1 if the guessing parameter of a 3PL model was set to a fixed value.
<code>eqmet</code>	the equating method used.
<code>DIFtype</code>	character indicating which parameters were tested for DIF.
<code>reference</code>	the reference group.
<code>focal</code>	the focal groups.
<code>names</code>	names of the groups.
<code>purification</code>	logical. If TRUE the purification procedure was applied.
<code>signif.level</code>	significance level used in the purification process
<code>equatings</code>	list containing the output of function <code>direc</code> .
<code>coef_trasf</code>	list containing the item parameters of each group transformed to the scale of the reference group.
<code>var_trasf</code>	list containing the covariance matrix of item parameters of each group transformed to the scale of the reference group.
<code>items.dif</code>	names of the items for which the null hypothesis of the test is rejected.
<code>anchor</code>	names of the items used as anchors.
<code>niter</code>	number of iterations.

### Author(s)

Michela Battauz

### References

- Battauz, M. (2018). On Wald tests for differential item functioning detection. *Statistical Methods and Applications*.
- Candell, G.L., Drasgow, F. (1988). An iterative procedure for linking metrics and assessing item bias in item response theory. *Applied Psychological Measurement*, **12**, 253-260.

### Examples

```
# load the data
data(dataDIF)
head(dataDIF)
# estimate a 2PL model for each group using the R package ltm
library(ltm)
data1 <- dataDIF[dataDIF$group == 1, 1:20]
data2 <- dataDIF[dataDIF$group == 2, 1:20]
```

```

data3 <- dataDIF[dataDIF$group == 3, 1:20]
mod1 <- ltm(data1 ~ z1)
mod2 <- ltm(data2 ~ z1)
mod3 <- ltm(data3 ~ z1)
# extract the coefficients and the covariance matrix
est1 <- import.ltm(mod1, display = FALSE)
est2 <- import.ltm(mod2, display = FALSE)
est3 <- import.ltm(mod3, display = FALSE)

# perform the test for DIF on two groups
res_diftest2 <- dif.test(coef = list(est1$coef, est2$coef),
  var = list(est1$var, est2$var))
res_diftest2

# perform the test for DIF on three groups
res_diftest3 <- dif.test(coef = list(est1$coef, est2$coef, est3$coef),
  var = list(est1$var, est2$var, est3$var))
res_diftest3

# perform the test for DIF on three groups
# reference group: 2
# equating method: Haebara
# purification applied
res_diftest3 <- dif.test(coef = list(est1$coef, est2$coef, est3$coef),
  var = list(est1$var, est2$var, est3$var), reference = 2,
  method = "Haebara", purification = TRUE)
res_diftest3

```

---

direc

*Direct Equating Coefficients*


---

## Description

Calculates direct equating coefficients and standard errors using IRT methods.

## Usage

```

direc(mods, which, mod1, mod2, method = "mean-mean", suff1 = ".1", suff2 = ".2",
  D = 1, quadrature = TRUE, nq = 30, items.select = NULL)

```

## Arguments

mods	an object of the class modIRT containing item parameter coefficients and their covariance matrix of the forms to be equated.
which	which forms to equate. Can be specified by name or number.
mod1	deprecated; please use mods instead. An object of the class modIRT containing item parameter coefficients and their covariance matrix of the first form.
mod2	deprecated; please use mods instead. An object of the class modIRT containing item parameter coefficients and their covariance matrix of the second form.

method	the equating method to be used. This should be one of "mean-mean", "mean-gmean", "mean-sigma", "Haebara" or "Stocking-Lord".
suff1	suffix to identify the first form to be equated.
suff2	suffix to identify the second form to be equated.
D	constant D of the IRT model used to estimate item parameters. See below for more details.
quadrature	logical; if TRUE the Gauss-Hermite quadrature is used to approximate the integral in the function that is minimized in the Haebara and Stocking-Lord methods. If FALSE the integral is replaced with a sum over 40 equally spaced values ranging from -4 to 4 with an increment of 0.05 and weights equal to one for all values.
nq	number of quadrature points used for the Gauss-Hermite quadrature if quadrature is TRUE.
items.select	optional character vector including the names of the items to use for equating.

### Details

Equating coefficients perform the conversion from the scale of the first form to the scale of the second form.

In the three-parameter logistic model the probability of a positive response on item  $i$  is

$$\pi_i = c_i + (1 - c_i) \frac{\exp[Da_i(\theta - b_i)]}{1 + \exp[Da_i(\theta - b_i)]}$$

where  $a_i$  is the item discrimination parameter,  $b_i$  is the item difficulty parameter,  $c_i$  is the item guessing parameter and  $\theta$  is the latent ability. The constant  $D$  can be specified using argument `D` of the `direc` function. The two-parameter logistic model can be obtained by setting  $c_i$  equal to zero, the one-parameter logistic model can be obtained by setting  $c_i$  equal to zero and  $a_i$  constant across items, while the Rasch model can be obtained by setting  $c_i$  equal to zero and  $a_i$  equal to 1.

The type of IRT model does not need to be specified as it is obtained from arguments `mod1` and `mod2`.

### Value

An object of class `eqc` with components

tab1	item parameters of the first form.
tab2	item parameters of the second form.
tab	Data frame containing item names ( <code>Item</code> ), item parameters of the first form (e.g. <code>test1</code> ), item parameters of the second form (e.g. <code>test2</code> ), and item parameters of the first form converted in the scale of the second form (e.g. <code>test1.as.test2</code> ).
var12	covariance matrix of item parameters of the first and the second form (only items used for equating).
varFull	list of covariance matrices of the item parameters (one matrix for each form)
partial	partial derivatives of equating coefficients A and B with respect to item parameters.

A	equating coefficient A.
B	equating coefficient B.
varAB	covariance matrix of the equating coefficients.
commonitem	list of length 2 containing the names of common item parameters and the names of the common items selected for equating.
suffixes	suffixes used to identify the forms.
ni	number of common items.
nis	number of common items selected for equating.
forms	names of equated forms.
method	the equating method used.
itmp	number of item parameters of the IRT model.

**Author(s)**

Michela Battauz

**References**

- Battauz, M. (2015). equateIRT: An R Package for IRT Test Equating. *Journal of Statistical Software*, **68**, 1–22.
- Kolen, M.J. and Brennan, R.L. (2014). *Test equating, scaling, and linking: methods and practices*, 3rd ed., New York: Springer
- Ogasawara, H. (2000). Asymptotic standard errors of IRT equating coefficients using moments. *Economic Review (Otaru University of Commerce)*, **51**, 1–23.
- Ogasawara, H. (2001). Standard Errors of Item Response Theory Equating/Linking by Response Function Methods. *Applied Psychological Measurement*, **25**, 53–67.

**See Also**

[eqc](#), [itm](#), [modIRT](#), [summary.eqc](#)

**Examples**

```
# three-parameter logistic model
# direct equating coefficients between forms 1 and 2 using the Stocking-Lord method
data(est3pl)
test <- paste("test", 1:5, sep = "")
mod3pl <- modIRT(coef = est3pl$coef, var = est3pl$var, names = test, display = FALSE)
l12 <- direc(mods = mod3pl, which = c("test1", "test2"), method = "Stocking-Lord")
summary(l12)

# two-parameter logistic model
# direct equating coefficients between forms 1 and 5 using the Haebara method
data(est2pl)
test <- paste("test", 1:5, sep = "")
mod2pl <- modIRT(coef = est2pl$coef, var = est2pl$var, names = test, display = FALSE)
l15 <- direc(mods = mod2pl, which = c(1,5), method = "Haebara")
```

```
summary(l15)

# Rasch model
# direct equating coefficients between forms 5 and 4 using the mean-mean method
data(estrasch)
test <- paste("test", 1:5, sep = "")
modrasch <- modIRT(coef = estrasch$coef, var = estrasch$var, names = test,
display = FALSE)
l54 <- direc(mods = modrasch, which = c(5,4), method = "mean-mean")
summary(l54)
```

---

eqc

*Extract Equating Coefficients*


---

## Description

eqc is a generic function which extracts the equating coefficients.

## Usage

```
eqc(x, ...)

## S3 method for class 'eqc'
eqc(x, ...)

## S3 method for class 'eqclist'
eqc(x, link = NULL, ...)

## S3 method for class 'ceqc'
eqc(x, ...)

## S3 method for class 'ceqclist'
eqc(x, link = NULL, path = NULL, ...)

## S3 method for class 'meqc'
eqc(x, link = NULL, path = NULL, ...)
```

## Arguments

x	object of the class eqc returned by function <a href="#">direc</a> or of the class eqclist returned by function <a href="#">alldirec</a> or of the class ceqc and ceqclist returned by function <a href="#">chaineq</a> or of the class meqc returned by function <a href="#">bisectorec</a> .
link	a character string with the names of the two forms being linked separated by a dot (e.g. "test1.test3").
path	a character string with the names of the forms that constitute the path separated by a dot (e.g. "test1.test2.test3").
...	further arguments passed to or from other methods.

**Value**

A data frame containing the equating coefficients for every link and path.

**Author(s)**

Michela Battauz

**See Also**

[alldirec](#), [bisectorec](#), [chainec](#), [direc](#)

**Examples**

```
# two-parameter logistic model
data(est2pl)
test<-paste("test", 1:5, sep = "")
mod2pl <- modIRT(coef = est2pl$coef, var = est2pl$var, names = test, display = FALSE)
# direct equating coefficients between forms 1 and 2 using the Haebara method
l12 <- direc(mods = mod2pl, which = c(1,2), method = "Haebara")
# all direct equating coefficients using the Haebara method
direclist2pl <- alldirec(mods = mod2pl, method = "Haebara")
# compute all chain equating coefficients of length 3
chainec3 <- chainec(r = 3, direclist = direclist2pl)
# compute chain equating coefficients for path 1,2,3,4
pth1 <- paste("test", 1:4, sep = "")
chainec1 <- chainec(direclist = direclist2pl, pths = pth1)
# compute chain equating coefficients for path 1,5,4
pth2 <- c(paste("test", c(1,5,4), sep = ""))
chainec2 <- chainec(direclist = direclist2pl, pths = pth2)
# create a list of objects of class ceqc
ecall <- c(chainec1, chainec2)
# compute bisector and weighted bisector coefficients
allec <- bisectorec(ecall = ecall, weighted = TRUE, unweighted = TRUE)

# extract equating coefficients
eqc(l12)
eqc(direclist2pl)
eqc(direclist2pl, link = "test1.test2")
eqc(chainec3)
eqc(chainec3, link = "test1.test3")
eqc(allec)
eqc(allec, path = "bisector")
```

**Description**

This dataset includes item parameter estimates and covariance matrices of a two-parameter logistic model applied to 5 simulated datasets with common items. The dichotomous item responses can be found in the dataset `data2pl`. See details for more information on the linkage plan.

**Usage**

```
data(est2pl)
```

**Format**

A list of length 2 with components:

`coef` a list of length 5 containing the matrices of item parameter estimates. Each matrix presents 2 columns; the first column contains difficulty parameters and the second column contains discrimination parameters. See details for information on the parameterization used. Names of rows correspond to the names of the items.

`var` a list of length 5 containing the covariance matrices of item parameter estimates.

**Details**

Every form is composed by 20 items and presents 10 items in common with adjacent forms. Furthermore, forms 1 and 5 present 10 common items. Use [linkp](#) to obtain a matrix with elements equal to the number of common items between different forms.

Item parameters are given under the parameterization used in the `ltm` package. Under this parameterization, the two-parameter logistic model is as follows

$$\pi_i = \frac{\exp(\beta_{1i} + \beta_{2i}z)}{1 + \exp(\beta_{1i} + \beta_{2i}z)},$$

where  $\pi_i$  denotes the conditional probability of responding correctly to the  $i$ th item given  $z$ ,  $\beta_{1i}$  is the easiness parameter,  $\beta_{2i}$  is the discrimination parameter, and  $z$  denotes the latent ability.

**Author(s)**

Michela Battauz

**See Also**

[data2pl](#), [linkp](#), [modIRT](#)

**Examples**

```
data(est2pl)
est2pl$coef
est2pl$var
linkp(coef = est2pl$coef)
```

est3pl

---

*Item Parameter Estimates and Covariance Matrices of a Three-Parameter Logistic Model*

---

**Description**

This dataset includes item parameter estimates and covariance matrices of a three-parameter logistic model applied to 5 simulated datasets with common items. See details for more information on the linkage plan.

**Usage**

```
data(est3pl)
```

**Format**

A list of length 2 with components:

`coef` a list of length 5 containing the matrices of item parameter estimates. Each matrix presents 3 columns; the first column contains guessing parameters, the second column contains difficulty parameters and the third column contains discrimination parameters. See details for information on the parameterization used. Names of rows correspond to the names of the items.

`var` a list of length 5 containing the covariance matrices of item parameter estimates.

**Details**

Every form is composed by 20 items and presents 10 items in common with adjacent forms. Furthermore, forms 1 and 5 present 10 common items. Use [linkp](#) to obtain a matrix with elements equal to the number of common items between different forms.

Item parameters are given under the parameterization used in the `ltm` package. Under this parameterization, the three-parameter logistic model is as follows

$$\pi_i = c_i + (1 - c_i) \frac{\exp(\beta_{1i} + \beta_{2i}z)}{1 + \exp(\beta_{1i} + \beta_{2i}z)},$$

where  $\pi_i$  denotes the conditional probability of responding correctly to the  $i$ th item given  $z$ ,  $c_i$  denotes the guessing parameter,  $\beta_{1i}$  is the easiness parameter,  $\beta_{2i}$  is the discrimination parameter, and  $z$  denotes the latent ability. Furthermore, the guessing parameters are given under this parameterization

$$c_i = \frac{\exp(c_i^*)}{1 + \exp(c_i^*)}.$$

**Author(s)**

Michela Battauz

**See Also**[linkp, modIRT](#)**Examples**

```
data(est3pl)
est3pl$coef
est3pl$var
linkp(coef = est3pl$coef)
```

estrasch

*Item Parameter Estimates and Covariance Matrices of a Rasch Model***Description**

This dataset includes item parameter estimates and covariance matrices of a Rasch model applied to 5 simulated datasets with common items. See details for more information on the linkage plan.

**Usage**

```
data(estrasch)
```

**Format**

A list of length 2 with components:

`coef` a list of length 5 containing the matrices of item parameter estimates. Each matrix presents 2 columns; the first column contains difficulty parameters and the second column is equal to 1. See details for information on the parameterization used. Names of rows correspond to the names of the items.

`var` a list of length 5 containing the covariance matrices of item parameter estimates.

**Details**

Every form is composed by 20 items and presents 10 items in common with adjacent forms. Furthermore, forms 1 and 5 present 10 common items. Use [linkp](#) to obtain a matrix with elements equal to the number of common items between different forms.

Item parameters are given under the parameterization used in the `ltm` package. Under this parameterization, the Rasch model is as follows

$$\pi_i = \frac{\exp(\beta_{1i} + z)}{1 + \exp(\beta_{1i} + z)},$$

where  $\pi_i$  denotes the conditional probability of responding correctly to the  $i$ th item given  $z$ ,  $\beta_{1i}$  is the easiness parameter, and  $z$  denotes the latent ability.

**Author(s)**

Michela Battauz

**See Also**

[linkp, modIRT](#)

**Examples**

```
data(estrasch)
estrasch$coef
estrasch$var
linkp(coef = estrasch$coef)
```

---

id.test

*Identity Test*

---

**Description**

Performs a statistical test to verify if the chain equating coefficients from one form to itself are  $A=1$  and  $B=0$ .

**Usage**

```
id.test(chain)
```

**Arguments**

chain            an object of the class ceqc returned by function [chainec](#).

**Value**

An object of class idtest with components

path

AB

statistic

df

p.value

**Author(s)**

Michela Battauz

**Examples**

```
# three-parameter logistic model
# direct equating coefficients using the "Haebara" method
data(est3pl)
test <- paste("test", 1:5, sep = "")
mod3pl <- modIRT(coef = est3pl$coef, var = est3pl$var, names = test, display = FALSE)
direclist3pl <- alldirec(mods = mod3pl, method = "Haebara")
# compute chain equating coefficients for path 1,2,3,4,5,1
pth3 <- paste("test", c(1:5,1), sep = "")
chainec_circle <- chainec(direclist = direclist3pl, pths = pth3)
summary(chainec_circle)
# test
id.test(chainec_circle)
```

import.ltm

*Import Item Parameters Estimates and Covariance Matrices from IRT Software*

**Description**

Import estimated item parameters and covariance matrix from the R packages ltm and mirt, and from external software IRTPRO and flexMIRT.

**Usage**

```
import.ltm(mod, display = TRUE, digits = 4)
```

```
import.mirt(mod, display = TRUE, digits = 3)
```

```
import.irtpro(fnamep, fnamev = NULL, fnameirt = NULL, display = TRUE, digits = 2)
```

```
import.flexmirt(fnamep, fnamev = NULL, fnameirt = NULL, display = TRUE, digits = 2)
```

**Arguments**

mod	output object from functions rasch, ltm, or tpm of the ltm package or from function mirt of the mirt package.
display	logical; if TRUE coefficients and standard errors are printed.
digits	integer indicating the number of decimal places to be used if display is TRUE.
fnamep	name of the file containing the estimated item parameters. Typically, -prm.txt.
fnamev	name of the file containing the covariance matrix of the estimated item parameters. Typically, -cov.txt.
fnameirt	name of the file containing additional information to link item parameters with the covariance matrix. Typically, -irt.txt.

**Details**

Item parameters are imported with the parameterization used by the software to estimate the IRT model. The usual IRT parameterization can be obtained later by using function [modIRT](#).

**Value**

A list with components

coef	item parameter estimates.
var	covariance matrix of item parameter estimates.

**Author(s)**

Michela Battauz

**References**

- Battauz, M. (2015). equateIRT: An R Package for IRT Test Equating. *Journal of Statistical Software*, **68**, 1–22.
- Cai L. (2013). *FlexMIRT version 2: Flexible Multilevel Multidimensional Item Analysis and Test Scoring [Computer Software]*. Chapel Hill, NC: Vector Psychometric Group.
- Cai, L., du Toit, S. H. C., Thissen, D. (2011). *IRTPRO: Flexible, multidimensional, multiple categorical IRT modeling [Computer software]*. Chicago: Scientific Software International.
- Chalmers, R. P. (2012). mirt: A Multidimensional Item Response Theory Package for the R Environment. *Journal of Statistical Software*, **48**, 1–29.
- Rizopoulos, D. (2006). ltm: an R package for latent variable modelling and item response theory analyses. *Journal of Statistical Software*, **17**, 1–25.

**See Also**

[modIRT](#)

**Examples**

```
#=====
# from package ltm
library(ltm)

# one-parameter logistic model
mod1pl <- rasch(LSAT)
est.mod1pl <- import.ltm(mod1pl)
est.mod1pl

# two-parameter logistic model
mod2pl <- ltm(LSAT ~ z1)
est.mod2pl <- import.ltm(mod2pl)
est.mod2pl
```

```

#####
# from package mirt
library(mirt)

# one-parameter logistic model
data(LSAT, package = "ltm")
val <- mirt(LSAT, 1, SE = TRUE, pars = "values")
cnstr <- val[val$name == "a1",]$parnum
mod1pl.m <- mirt(LSAT, 1, SE = TRUE, SE.type = 'Richardson', constrain = list(cnstr))
est.mod1pl.m <- import.mirt(mod1pl.m, digits = 4)
est.mod1pl.m

# two-parameter logistic model
data(LSAT, package = "ltm")
mod2pl.m <- mirt(LSAT, 1, SE = TRUE, SE.type = 'Richardson')
est.mod2pl.m <- import.mirt(mod2pl.m, display = FALSE)
est.mod2pl.m

```

itm

*Extract Item Parameters***Description**

`itm` is a generic function which extracts a data frame containing the item parameters of two forms being equated in the original scale and item parameters of the first form converted in the scale of the second form.

**Usage**

```

itm(x, ...)

## S3 method for class 'eqc'
itm(x, ...)

## S3 method for class 'eqclist'
itm(x, link, ...)

## S3 method for class 'ceqc'
itm(x, ...)

## S3 method for class 'ceqclist'
itm(x, path, ...)

## S3 method for class 'meqc'
itm(x, link, bitype = NULL, ...)

```

**Arguments**

x	object of the class <code>eqc</code> returned by function <code>direc</code> or of the class <code>eqclist</code> returned by function <code>alldirec</code> or of the class <code>ceqc</code> and <code>ceqclist</code> returned by function <code>chainec</code> or of the class <code>meqc</code> returned by function <code>bisectorec</code> .
link	a character string with the names of the two forms being linked separated by a dot (e.g. "test1.test3").
path	a character string with the names of the forms that constitute the path separated by a dot (e.g. "test1.test2.test3").
bistype	bisector type. It should be weighted or unweighted. It could be NULL if only one type was computed by function <code>bisectorec</code> .
...	further arguments passed to or from other methods.

**Value**

A data frame containing item names (`Item`), item parameters of the first form (e.g. `test1`), item parameters of the last form (e.g. `test3`), and item parameters of the first form converted in the scale of the last form (e.g. `test1.as.test3`).

**Author(s)**

Michela Battauz

**See Also**

[convert](#), [alldirec](#), [bisectorec](#), [chainec](#), [direc](#)

**Examples**

```
# two-parameter logistic model
data(est2pl)
test<-paste("test", 1:5, sep = "")
mod2pl <- modIRT(coef = est2pl$coef, var = est2pl$var, names = test, display = FALSE)
# direct equating coefficients between forms 1 and 2 using the Haebara method
l12 <- direc(mods = mod2pl, which = c(1,2), method = "Haebara")
# all direct equating coefficients using the Haebara method
direclist2pl <- alldirec(mods = mod2pl, method = "Haebara")
# compute all chain equating coefficients of length 3
chainec3 <- chainec(r = 3, direclist = direclist2pl)
# compute chain equating coefficients for path 1,2,3,4
pth1 <- paste("test", 1:4, sep = "")
chainec1 <- chainec(direclist = direclist2pl, pths = pth1)
# compute chain equating coefficients for path 1,5,4
pth2 <- c(paste("test", c(1,5,4), sep = ""))
chainec2 <- chainec(direclist = direclist2pl, pths = pth2)
# create a list of objects of class ceqc
ecall <- c(chainec1, chainec2)
# compute bisector and weighted bisector coefficients
allec <- bisectorec(ecall = ecall, weighted = TRUE, unweighted = TRUE)
```

```
# extract item parameters
itm(l12)
itm(direclist2pl, link = "test1.test2")
itm(chainec3, path = "test1.test2.test3")
itm(allec, bitype = "weighted")
```

---

linkp

*Linkage Plan*

---

### Description

Calculates the number of common items between a list of forms.

### Usage

```
linkp(coef)
```

### Arguments

coef            list of matrices (one for each form) containing item parameter estimates. The names of the rows of each matrix should be the names of the items.

### Value

A matrix whose elements indicate the number of common items between the forms. On the diagonal of the matrix there are the number of items of each form.

### Author(s)

Michela Battauz

### See Also

[est2pl](#), [est3pl](#), [estrasch](#)

### Examples

```
data(est3pl)
linkp(coef = est3pl$coef)
```

modIRT

*Estimated Coefficients and Covariance Matrix of IRT Models***Description**

Creates an object of the class `modIRT` containing estimated coefficients and covariance matrices of IRT models. Rasch, one-parameter logistic, two-parameter logistic and three-parameter logistic models are included.

**Usage**

```
modIRT(coef, var = NULL, names = NULL, ltparam = TRUE, lparam = TRUE,
display = TRUE, digits = 2)
```

**Arguments**

<code>coef</code>	list of matrices (one for each form) containing item parameter estimates. Guessing, difficulty and discrimination parameters should strictly be given in this order and they are contained in different columns of the matrix. The names of the rows of each matrix should be the names of the items.
<code>var</code>	list of matrices (one for each form) containing the covariance matrix of item parameter estimates. They should be given in the same order of coefficients.
<code>names</code>	character vector containing the names of the forms. This should have the same length of <code>coef</code> and <code>var</code> . If <code>NULL</code> , the names of the forms are assigned by function <code>modIRT</code> .
<code>ltparam</code>	logical; if <code>TRUE</code> the latent trait parameterization is used for difficulty parameters and the <code>modIRT</code> function performs a transformation of item parameters to return them in the usual IRT parameterization. Set to <code>FALSE</code> to avoid transformations. See below for more details.
<code>lparam</code>	logical; if <code>TRUE</code> the logistic parameterization is used for guessing parameters and the <code>modIRT</code> function performs a transformation of item parameters to return them in the usual IRT parameterization. Set to <code>FALSE</code> to avoid transformations. See below for more details.
<code>display</code>	logical; if <code>TRUE</code> coefficients and standard errors are printed.
<code>digits</code>	integer indicating the number of decimal places to be used if <code>display</code> is <code>TRUE</code> .

**Details**

`ltparam` and `lparam` refers the the parameterization used by the software used to estimate item parameters. The R package `ltm`, and the programs `IRTPRO` and `flexMIRT` use these parameterizations. If `ltparam` is `TRUE` the latent trait parameterization is used. Under this parameterization, the three-parameter logistic model is as follows

$$\pi_i = c_i + (1 - c_i) \frac{\exp(\beta_{1i} + \beta_{2i}z)}{1 + \exp(\beta_{1i} + \beta_{2i}z)},$$

where  $\pi_i$  denotes the conditional probability of responding correctly to the  $i$ th item given  $z$ ,  $c_i$  denotes the guessing parameter,  $\beta_{1i}$  is the easiness parameter,  $\beta_{2i}$  is the discrimination parameter, and  $z$  denotes the latent ability. The two-parameter logistic model, the one-parameter logistic model and the Rasch model present the same formulation. The two-parameter logistic model can be obtained by setting  $c_i$  equal to zero, the one-parameter logistic model can be obtained by setting  $c_i$  equal to zero and  $\beta_{2i}$  constant across items, while the Rasch model can be obtained by setting  $c_i$  equal to zero and  $\beta_{2i}$  equal to 1.

If `lparam` is TRUE the guessing parameters are given under this parameterization

$$c_i = \frac{\exp(c_i^*)}{1 + \exp(c_i^*)}.$$

The `modIRT` function returns parameter estimates under the usual IRT parameterization, that is,

$$\pi_i = c_i + (1 - c_i) \frac{\exp[Da_i(\theta - b_i)]}{1 + \exp[Da_i(\theta - b_i)]},$$

where  $Da_i = \beta_{2i}$ ,  $b_i = -\beta_{1i}/\beta_{2i}$  and  $\theta = z$ .

If `ltparam` or `lparam` are TRUE, the covariance matrix is calculated using the delta method.

If item parameters are already given under the usual IRT parameterization, arguments `ltparam` and `lparam` should be set to FALSE.

### Value

An object of class `modIRT` consisting in a list with length equal to the number of forms containing lists with components

<code>coefficients</code>	item parameter estimates.
<code>var</code>	covariance matrix of item parameter estimates.
<code>itmp</code>	number of item parameters of the IRT model. This is 1 for the Rasch model, 2 for the one-parameter logistic model with constant discriminations, 2 for the two-parameter logistic model and 3 for the three-parameter logistic model.

### Author(s)

Michela Battauz

### References

- Battauz, M. (2015). `equateIRT`: An R Package for IRT Test Equating. *Journal of Statistical Software*, **68**, 1–22.
- Bartholomew, D., Knott, M. and Moustaki, I. (2011) *Latent Variable Models and Factor Analysis: a Unified Approach*, 3rd ed. Wiley.
- Rizopoulos, D. (2006). `ltm`: an R package for latent variable modelling and item response theory analyses. *Journal of Statistical Software*, **17**, 1–25.

### See Also

[direc](#), [import.ltm](#)

**Examples**

```

# three-parameter logistic model
data(est3pl)
test <- paste("test", 1:5, sep = "")
mod3pl <- modIRT(coef = est3pl$coef, var = est3pl$var, names = test, display = FALSE)

# two-parameter logistic model
data(est2pl)
test <- paste("test", 1:5, sep = "")
mod2pl <- modIRT(coef = est2pl$coef, var = est2pl$var, names = test, display = FALSE)

# Rasch model
data(estrasch)
test <- paste("test", 1:5, sep = "")
modrasch <- modIRT(coef = estrasch$coef, var = estrasch$var, names = test,
display = FALSE)

# one-parameter logistic model imported from the R package ltm
library(ltm)
mod1pl <- rasch(LSAT)
summary(mod1pl)
est.mod1pl <- import.ltm(mod1pl)
mod1pl.ltm <- modIRT(coef = list(est.mod1pl$coef), var = list(est.mod1pl$var), digits = 4)

```

---

score

*Scoring*


---

**Description**

Relates number-correct scores on two forms.

**Usage**

```

score(obj, ...)

## S3 method for class 'eqc'
score(obj, method = "TSE", D = 1, scores = NULL, se = TRUE, nq = 30,
      w = 0.5, theta = NULL, weights = NULL, ...)

## S3 method for class 'eqclist'
score(obj, link = NULL, method = "TSE", D = 1, scores = NULL, se = TRUE, nq = 30,
      w = 0.5, theta = NULL, weights = NULL, ...)

## S3 method for class 'ceqc'
score(obj, method = "TSE", D = 1, scores = NULL, se = TRUE, nq = 30,
      w = 0.5, theta = NULL, weights = NULL, ...)

```

```
## S3 method for class 'ceqclist'
score(obj, path = NULL, method = "TSE", D = 1, scores = NULL, se = TRUE, nq = 30,
      w = 0.5, theta = NULL, weights = NULL, ...)

## S3 method for class 'meqc'
score(obj, link = NULL, method = "TSE", D = 1, scores = NULL, se = TRUE,
      bitype = NULL, nq = 30, w = 0.5, theta = NULL, weights = NULL, ...)
```

### Arguments

obj	output object from functions <a href="#">direc</a> , <a href="#">alldirec</a> , <a href="#">chaineq</a> or <a href="#">bisectorec</a> .
link	a character string with the names of the two forms being linked separated by a dot (e.g. "test1.test3"). Necessary if obj is output of functions <a href="#">alldirec</a> or <a href="#">bisectorec</a> .
path	a character string with the names of the forms that constitute the path separated by a dot (e.g. "test1.test2.test3"). Necessary if obj is output of function <a href="#">chaineq</a> .
method	the scoring method to be used. This should be one of "TSE" (the default) for true score equating or "OSE" for observed score equating.
D	constant D of the IRT model used to estimate item parameters.
scores	integer values to be converted.
se	logical; is TRUE standard errors of equated scores are computed.
bitype	bisector type. It should be specified when obj is an output object from function <a href="#">bisectorec</a> . It should be weighted or unweighted. It can be NULL if only one type was computed by function <a href="#">bisectorec</a> .
nq	number of quadrature points used to approximate integrals with observed score equating. Used only if arguments theta and weights are NULL.
w	synthetic weight for population 1. It should be a number between 0 and 1.
theta	vector of ability values used to approximate integrals with observed score equating.
weights	vector of weights used to approximate integrals with observed score equating.
...	further arguments passed to or from other methods.

### Details

In this function common items are internal, i.e. they are used for scoring the test.

This function computes standard error of equated scores with the observed score equating method using  $(\alpha'_X, \alpha'_{V1}, A, B)'$  for the description of  $\beta_{X2,x}(At_m + B)$ , and  $(\alpha'_Y, \alpha'_{V2}, A, B)'$  for  $\beta_{Y1,y}(t_m)$  (see Ogasawara, 2003).

### Value

A data frame containing theta values (only for true score equating), scores in the form chosen as base, equated scores, and standard errors of equated scores.

**Author(s)**

Michela Battauz

**References**

- Kolen, M.J. and Brennan, R.L. (2014). *Test equating, scaling, and linking: methods and practices*, 3rd ed., New York: Springer.
- Ogasawara, H. (2001). Item response theory true score equatings and their standard errors. *Journal of Educational and Behavioral Statistics*, **26**, 31–50.
- Ogasawara, H. (2003). Asymptotic standard errors of IRT observed-score equating methods. *Psychometrika*, **68**, 193–211.

**See Also**

[alldirec](#), [bisectorec](#), [chainec](#), [direc](#)

**Examples**

```
# two-parameter logistic model
data(est2pl)
test<-paste("test", 1:5, sep = "")
mod2pl <- modIRT(coef = est2pl$coef, var = est2pl$var, names = test, display = FALSE)
# direct equating coefficients between forms 1 and 2 using the Haebara method
l12 <- direc(mods = mod2pl, which = c(1,2), method = "Haebara")
# scoring using direct equating coefficients and the true score equating method
score(l12)
# scoring using direct equating coefficients and the observed score equating method
score(l12, method = "OSE")
# specify only scores from 10 to 15
score(l12, method = "OSE", scores = 10:15)

# all direct equating coefficients using the Haebara method
direclist2pl <- alldirec(mods = mod2pl, method = "Haebara")
# scoring using direct equating coefficients and the true score equating method
score(direclist2pl, link = "test1.test2")

# compute chain equating coefficients for path 1,2,3,4
pth1 <- paste("test", 1:4, sep = "")
chainec1 <- chainec(direclist = direclist2pl, pths = pth1)
# compute chain equating coefficients for path 1,5,4
pth2 <- paste("test", c(1,5,4), sep = "")
chainec2 <- chainec(direclist = direclist2pl, pths = pth2)
# scoring using chain equating coefficients and the true score equating method for score 12
score(chainec1, path = "test1.test2.test3.test4", scores = 12)
score(chainec2, path = "test1.test5.test4", scores = 12)

# create a list of objects of class ceqc
ecall <- c(chainec1, chainec2)
# compute bisector and weighted bisector coefficients
allec <- bisectorec(ecall = ecall, weighted = TRUE, unweighted = TRUE)
# scoring using weighted bisector equating coefficients and the true score
```

```
# equating method for score 12
score(allec, bitype = "weighted", link = "test1.test4", scores = 12)
```

---

sd.test

*Test on Chain Equating Coefficients*


---

### Description

Performs a statistical test to verify if the chain equating coefficients that link the same two forms are equal.

### Usage

```
sd.test(ecall)
```

### Arguments

ecall            list of objects of class eqc or ceqc returned by functions [direc](#) and [chainec](#).

### Value

An object of class idtest with components

```
link
paths
AB
statistic
df
p.value
```

### Author(s)

Michela Battauz

### Examples

```
# three-parameter logistic model
# direct equating coefficients using the "Haebara" method
data(est3pl)
test <- paste("test", 1:5, sep = "")
mod3pl <- modIRT(coef = est3pl$coef, var = est3pl$var, names = test, display = FALSE)
direclist3pl <- alldirec(mods = mod3pl, method = "Haebara")
# compute chain equating coefficients for path 1,2,3,4,5
pth3 <- paste("test", 1:5, sep = "")
chainec3 <- chainec(direclist = direclist3pl, pths = pth3)
# create a list of objects of class eqc or ceqc
ecall <- c(chainec3, direclist3pl["test1.test5"])
summary(chainec3)
```

```
summary(direclist3pl$test1.test5)
# test
sd.test(ecall)
```

---

summary.ceqc

*Summarizing Estimated Chain Equating Coefficients*


---

## Description

summary method for class ceqc.

## Usage

```
## S3 method for class 'ceqc'
summary(object, ...)

## S3 method for class 'summary.ceqc'
print(x, ...)
```

## Arguments

object	an object of the class ceqc returned by function <a href="#">chainec</a> .
x	an object of class summary.ceqc, a result of a call to summary.ceqc.
...	further arguments passed to or from other methods.

## Value

The function summary.ceqc returns a list with components

forms	names of equated forms.
method	the equating method used.
coefficients	a $2 \times 2$ matrix with columns for the estimated coefficients A and B and standard errors.

## Author(s)

Michela Battauz

## See Also

[chainec](#)

**Examples**

```
# two-parameter logistic model
# direct equating coefficients using the "Haebara" method
data(est2pl)
test<-paste("test", 1:5, sep = "")
mod2pl <- modIRT(coef = est2pl$coef, var = est2pl$var, names = test, display = FALSE)
direclist2pl <- alldirec(mods = mod2pl, method = "Haebara")
# compute all chain equating coefficients of length 3
chainec3 <- chainec(r = 3, direclist = direclist2pl)
summary(chainec3$test1.test5.test4)
```

summary.ceqclist

*Summarizing a List of Estimated Chain Equating Coefficients***Description**

summary method for class ceqclist.

**Usage**

```
## S3 method for class 'ceqclist'
summary(object, path, ...)

## S3 method for class 'summary.ceqclist'
print(x, ...)
```

**Arguments**

object	an object of the class ceqclist returned by function <a href="#">chainec</a> .
path	a vector of character strings with the names of the forms that constitute the path separated by a dot (e.g. "test1.test2.test3").
x	an object of class summary.ceqclist, a result of a call to summary.ceqclist.
...	further arguments passed to or from other methods.

**Value**

The function `summary.ceqclist` returns a list containing the output of function [summary.ceqc](#) for each path contained in object.

**Author(s)**

Michela Battauz

**See Also**

[chainec](#), [summary.eqc](#)

**Examples**

```
# two-parameter logistic model
# direct equating coefficients using the "Haebara" method
data(est2pl)
test<-paste("test", 1:5, sep = "")
mod2pl <- modIRT(coef = est2pl$coef, var = est2pl$var, names = test, display = FALSE)
direclist2pl <- alldirec(mods = mod2pl, method = "Haebara")
# compute all chain equating coefficients of length 3
chainec3 <- chainec(r = 3, direclist = direclist2pl)
summary(chainec3)
summary(chainec3, path = "test1.test2.test3")
summary(chainec3, path = c("test1.test2.test3","test1.test5.test4"))
```

summary.eqc

*Summarizing Estimated Equating Coefficients***Description**

summary method for class eqc.

**Usage**

```
## S3 method for class 'eqc'
summary(object, ...)

## S3 method for class 'summary.eqc'
print(x, ...)
```

**Arguments**

object	an object of the class eqc returned by function <a href="#">direc</a> .
x	an object of class summary.eqc, a result of a call to summary.eqc.
...	further arguments passed to or from other methods.

**Value**

The function summary.eqc returns a list with components

forms	names of equated forms.
method	the equating method used.
coefficients	a $2 \times 2$ matrix with columns for the estimated coefficients A and B and standard errors.

**Author(s)**

Michela Battauz

**See Also**[direc](#)**Examples**

```
# three-parameter logistic model
# direct equating coefficients between forms 1 and 2 using the Stocking-Lord method
data(est3pl)
test <- paste("test", 1:5, sep = "")
mod3pl <- modIRT(coef = est3pl$coef, var = est3pl$var, names = test, display = FALSE)
l12 <- direc(mod1 = mod3pl[1], mod2 = mod3pl[2], method = "Stocking-Lord")
summary(l12)
```

summary.eqclist

*Summarizing a List of Estimated Direct Equating Coefficients***Description**

summary method for class eqclist.

**Usage**

```
## S3 method for class 'eqclist'
summary(object, link, ...)

## S3 method for class 'summary.eqclist'
print(x, ...)
```

**Arguments**

object	an object of the class eqclist returned by function <a href="#">alldirec</a> .
link	a vector of character strings with the names of the two forms being linked separated by a dot (e.g. "test1.test2").
x	an object of class summary.eqclist, a result of a call to summary.eqclist.
...	further arguments passed to or from other methods.

**Value**

The function summary.eqclist returns a list containing the output of function [summary.eqc](#) for each link contained in object.

**Author(s)**

Michela Battauz

**See Also**

[alldirec](#), [summary.eqc](#)

**Examples**

```
# Rasch model
# direct equating coefficients using the "mean-mean" method
data(estrasc)
test<-paste("test", 1:5, sep = "")
modrasch <- modIRT(coef = estrasc$coef, var = estrasc$var, names = test,
display = FALSE)
direclistrasc <- alldirec(mods = modrasch, method = "mean-mean", all = TRUE)
summary(direclistrasc)
summary(direclistrasc, link = "test1.test2")
summary(direclistrasc, link = c("test1.test2", "test1.test5"))
```

---

summary.meqc

*Summarizing Bisector Equating Coefficients*


---

**Description**

summary method for class meqc.

**Usage**

```
## S3 method for class 'meqc'
summary(object, ...)

## S3 method for class 'summary.meqc'
print(x, ...)
```

**Arguments**

object	an object of the class meqc returned by function <a href="#">bisectorec</a> .
x	an object of class summary.meqc, a result of a call to summary.meqc.
...	further arguments passed to or from other methods.

**Value**

The function summary.meqc returns a list with components

link	character vector with names of equated forms.
method	the equating method used.
coefficients	list of data frames containing Path, Estimate and StdErr of direct, chain and bisector equating coefficients.

**Author(s)**

Michela Battauz

**See Also**[bisectorec](#)**Examples**

```
# three-parameter logistic model
# direct equating coefficients using the "Stocking-Lord" method
data(est3pl)
test <- paste("test", 1:5, sep = "")
mod3pl <- modIRT(coef = est3pl$coef, var = est3pl$var, names = test, display = FALSE)
direclist3pl <- alldirec(mods = mod3pl, method = "Stocking-Lord")
# compute chain equating coefficients for path 1,2,3,4
pth1 <- paste("test", 1:4, sep = "")
pth1 <- data.frame(t(pth1), stringsAsFactors = FALSE)
chainec1 <- chainec(direclist = direclist3pl, pths = pth1)
# compute chain equating coefficients for path 1,5,4
pth2 <- c(paste("test", c(1,5,4), sep = ""))
pth2 <- data.frame(t(pth2), stringsAsFactors = FALSE)
chainec2 <- chainec(direclist = direclist3pl, pths = pth2)
# compute chain equating coefficients for path 1,2,3,4,5
pth3 <- paste("test", 1:5, sep = "")
pth3 <- data.frame(t(pth3), stringsAsFactors = FALSE)
chainec3 <- chainec(direclist = direclist3pl, pths = pth3)
# create a list of objects of class eqc or ceqc
ecall <- c(chainec1, chainec2, chainec3, direclist3pl["test1.test5"])
# compute bisector and weighted bisector coefficients
allec <- bisectorec(ecall = ecall, weighted = TRUE, unweighted = TRUE)
summary(allec)
```

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