

Package ‘stabledist’

September 12, 2016

Version 0.7-1

Date 2016-09-12

Title Stable Distribution Functions

Author Diethelm Wuertz, Martin Maechler and Rmetrics core team members.

Maintainer Martin Maechler <maechler@stat.math.ethz.ch>

Depends R (>= 3.1.0)

Imports stats

Suggests Matrix, fBasics, FMStable, RUnit, Rmpfr, sfsmisc

Description Density, Probability and Quantile functions, and random number generation for (skew) stable distributions, using the parametrizations of Nolan.

LazyData yes

License GPL (>= 2)

URL <http://www.rmetrics.org>,
<https://r-forge.r-project.org/scm/viewvc.php/pkg/stabledist/?root=rmetrics>

NeedsCompilation no

Repository CRAN

Date/Publication 2016-09-12 20:55:10

R topics documented:

| | |
|------------------------------|----------|
| StableDistribution | 2 |
| StableMode | 5 |
| Index | 7 |

Description

A collection and description of functions to compute density, distribution and quantile function and to generate random variates of the stable distribution.

The four functions are:

[dpqr]stable the (skewed) stable distribution.

Usage

```
dstable(x, alpha, beta, gamma = 1, delta = 0, pm = 0,
        log = FALSE,
        tol = 64*.Machine$double.eps, zeta.tol = NULL,
        subdivisions = 1000)
pstable(q, alpha, beta, gamma = 1, delta = 0, pm = 0,
        lower.tail = TRUE, log.p = FALSE, silent = FALSE,
        tol = 64*.Machine$double.eps, subdivisions = 1000)
qstable(p, alpha, beta, gamma = 1, delta = 0, pm = 0,
        lower.tail = TRUE, log.p = FALSE,
        tol = .Machine$double.eps^0.25, maxiter = 1000, trace = 0,
        integ.tol = 1e-7, subdivisions = 200)
rstable(n, alpha, beta, gamma = 1, delta = 0, pm = 0)
```

Arguments

| | |
|---------------------------|---|
| alpha, beta, gamma, delta | value of the index parameter alpha in the interval $(0, 2]$; skewness parameter beta, in the range $[-1, 1]$; scale parameter gamma; and location (or 'shift') parameter delta. |
| n | sample size (integer). |
| p | numeric vector of probabilities. |
| pm | parameterization, an integer in $\{0, 1, 2\}$; by default pm=0, the 'S0' parameterization. |
| x, q | numeric vector of quantiles. |
| log, log.p | logical; if TRUE, probabilities p are given as $\log(p)$. |
| lower.tail | logical; if TRUE (default), probabilities are $P[X \leq x]$ otherwise, $P[X > x]$. |
| silent | logical indicating that e.g., warnings should be suppressed when NaN is produced (because of numerical problems). |
| integ.tol | positive number, the tolerance used for numerical integration, see integrate . |
| tol | numerical tolerance, |

| | |
|-----------------------------|--|
| | dstable(), pstable(): used for numerical integration, see <code>integ.tol</code> above. Note that earlier versions had tighter tolerances – which seem too tight as default values. |
| | qstable(): used for rootfinding, see <code>uniroot</code> . |
| <code>zeta.tol</code> | (<code>dstable</code>) numerical tolerance for checking if x is close to $\zeta(\alpha, \beta)$. The default, <code>NULL</code> depends itself on (α, β) . As it is <i>experimental</i> and not guaranteed to remain in the future, its use is not recommended in production code. Rather e-mail the package maintainer about it. |
| <code>subdivisions</code> | maximal number of intervals for integration, see <code>integrate</code> . |
| <code>maxiter, trace</code> | maximal number of iterations and verbosity in <code>uniroot</code> , see there. |

Details

Skew Stable Distribution:

The function uses the approach of J.P. Nolan for general stable distributions. Nolan (1997) derived expressions in form of integrals based on the characteristic function for standardized stable random variables. For `dstable` and `pstable`, these integrals are numerically evaluated using R's `integrate()` function.

“S0” parameterization [`pm=0`]: based on the (M) representation of Zolotarev for an alpha stable distribution with skewness beta. Unlike the Zolotarev (M) parameterization, gamma and delta are straightforward scale and shift parameters. This representation is continuous in all 4 parameters, and gives an intuitive meaning to gamma and delta that is lacking in other parameterizations.

Switching the sign of beta *mirrors* the distribution at the vertical axis $x = \delta$, i.e.,

$$f(x, \alpha, -\beta, \gamma, \delta, 0) = f(2\delta - x, \alpha, +\beta, \gamma, \delta, 0),$$

see the graphical example below.

“S” or “S1” parameterization [`pm=1`]: the parameterization used by Samorodnitsky and Taqqu in the book *Stable Non-Gaussian Random Processes*. It is a slight modification of Zolotarev’s (A) parameterization.

“S*” or “S2” parameterization [`pm=2`]: a modification of the S0 parameterization which is defined so that (i) the scale gamma agrees with the Gaussian scale (standard dev.) when $\alpha=2$ and the Cauchy scale when $\alpha=1$, (ii) the mode is exactly at delta. For this parametrization, `stableMode(alpha, beta)` is needed.

“S3” parameterization [`pm=3`]: an internal parameterization, currently not available for these functions. The scale is the same as the “S2” parameterization, the shift is $-\beta * g(\alpha)$, where $g(\alpha)$ is defined in Nolan(1999).

Value

All values for the `*stable` functions are numeric vectors: `d*` returns the density, `p*` returns the distribution function, `q*` returns the quantile function, and `r*` generates random deviates.

Tail Behavior

The asymptotic behavior for large x , aka “tail behavior” for the cumulative $F(x) = P(X \leq x)$ is (for $x \rightarrow \infty$)

$$1 - F(x) \sim (1 + \beta)C_\alpha x^{-\alpha},$$

where $C_\alpha = \Gamma(\alpha)/\pi \sin(\alpha\pi/2)$; hence also

$$F(-x) \sim (1 - \beta)C_\alpha x^{-\alpha}.$$

Differentiating $F()$ above gives

$$f(x) \sim \alpha(1 + \beta)C_\alpha x^{-(1+\alpha)}.$$

Note

In the case $\beta = 1$, the distributions are “maximally skewed to the right” or simply “*extremal stable*” (Zolotarev). In that case, the package **FMStable** provides `dpq*` functions which are faster and more accurate than ours (if accuracy higher than about 6 digits is needed), see, [pEstable](#).

When alpha is close to 1 or close to 0 (“close”, e.g., meaning distance $d < 0.01$), the computations typically are numerically considerably more challenging, and the results may not be accurate.

As we plan to improve on this, *and* as it is unknown when exactly the numerical difficulties arise, we currently only do warn here (in the documentation), but *not* by giving explicit `warning()`s.

Author(s)

Diethelm Wuertz for the original Rmetrics R-port. Many numerical improvements by Martin Maechler.

References

Chambers J.M., Mallows, C.L. and Stuck, B.W. (1976) *A Method for Simulating Stable Random Variables*, J. Amer. Statist. Assoc. **71**, 340–344.

John P. Nolan (2012) *Stable Distributions - Models for Heavy Tailed Data* Birkhauser, Boston; in progress, chapter 1 online at <http://academic2.american.edu/~jpnolan/stable/chap1.pdf>

Nolan J.P. (1997) Numerical calculation of stable densities and distribution functions. *Stochastic Models* **13**(4), 759–774.

Also available as ‘density.ps’ from Nolan’s web page.

Samoridnitsky G., Taqqu M.S. (1994); *Stable Non-Gaussian Random Processes, Stochastic Models with Infinite Variance*, Chapman and Hall, New York, 632 pages.

Weron, A., Weron R. (1999); *Computer Simulation of Levy alpha-Stable Variables and Processes*, Preprint Technical University of Wroclaw, 13 pages.

See Also

the `stableSlider()` function from package **fBasics** for displaying densities and probabilities of these distributions, for educational purposes.

Examples

```
## stable -

## Plot stable random number series
set.seed(1953)
r <- rstable(n = 1000, alpha = 1.9, beta = 0.3)
```

```

plot(r, type = "l", main = "stable: alpha=1.9 beta=0.3",
     col = "steelblue")
grid()

## Plot empirical density and compare with true density:
hist(r, n = 25, probability = TRUE, border = "white",
     col = "steelblue")
x <- seq(-5, 5, 0.25)
lines(x, dstable(x, alpha = 1.9, beta = 0.3, tol= 1e-3), lwd = 2)

## Plot df and compare with true df:
plot(ecdf(r), do.points=TRUE, col = "steelblue",
     main = "Probabilities:  ecdf(rstable(1000,..)) and true  cdf F()")
rug(r)
lines(x, pstable(q = x, alpha = 1.9, beta = 0.3),
     col="#0000FF88", lwd= 2.5)

## Switching sign(beta) <=> Mirror the distribution around x == delta:
curve(dstable(x, alpha=1.2, beta = .8, gamma = 3, delta = 2), -10, 10)
curve(dstable(x, alpha=1.2, beta = -.8, gamma = 3, delta = 2),
     add=TRUE, col=2)
## or the same
curve(dstable(2*2-x, alpha=1.2, beta = +.8, gamma = 3, delta = 2),
     add=TRUE, col=adjustcolor("gray",0.2), lwd=5)
abline(v = 2, col = "gray", lty=2, lwd=2)
axis(1, at = 2, label = expression(delta == 2))

## Compute quantiles:
x. <- -4:4
px <- pstable(x., alpha = 1.9, beta = 0.3)
(qs <- qstable(px, alpha = 1.9, beta = 0.3))
stopifnot(all.equal(as.vector(qs), x., tol = 1e-5))

```

StableMode

Mode of the Stable Distribution Function

Description

Computes the mode of the stable distribution, i.e., the maximum of its density function in the "0" parametrization, i.e., the maximum x_0 of `dstable(x, alpha, beta, gamma = 1, delta = 0, pm = 0)`.

Finds the maximum of `dstable` numerically, using `optimize`.

Usage

```

stableMode(alpha, beta,
           beta.max = 1 - 1e-11,
           tol = .Machine$double.eps^0.25)

```

Arguments

| | |
|-------------|--|
| alpha, beta | numeric parameters: value of the index parameter alpha in the range $(0, 2]$, and the skewness parameter beta, in the range $[-1, 1]$. |
| beta.max | for numerical purposes, values of beta too close to 1, are set to beta.max. Do not modify unless you know what you're doing. |
| tol | numerical tolerance for <code>optimize()</code> . |

Value

returns a numeric value, the location of the stable mode.

Author(s)

Diethelm Wuertz for the Rmetrics R-port; minor cleanup by Martin Maechler.

See Also

For definition and the “dpqr”-functions, [StableDistribution](#), also for the references.

Examples

```
## beta = 0 <==> symmetric <==> mode = 0
all.equal(stableMode(alpha=1, beta=0), 0)
al.s <- c(1e-100, seq(0,2, by = 1/32)[-1])
stopifnot(vapply(al.s, function(alp)
  stableMode(alpha=alp, beta=0), 1.) == 0)

## more interesting: asymmetric (beta != 0):
stableMode(alpha=1.2, beta=0.1)

if(stabledist:::doExtras()) { # takes 2.5 seconds
  sm0.5 <- vapply(al.s, function(AA)
    stableMode(alpha=AA, beta= 0.5), 1.)
  plot(al.s, sm0.5, type = "o", col=2, xlab = quote(alpha), ylab="mode",
    main = quote("Mode of stable"*{ }(alpha, beta == 0.5, pm==0))
  }
}
```

Index

*Topic **distribution**

StableDistribution, 2

StableMode, 5

dstable, 5

dstable (StableDistribution), 2

integrate, 2, 3

optimize, 5, 6

pEstable, 4

pstable (StableDistribution), 2

qstable (StableDistribution), 2

rstable (StableDistribution), 2

StableDistribution, 2, 6

StableMode, 5

stableMode, 3

stableMode (StableMode), 5

stableSlider, 4

uniroot, 3

warning, 4